UCSD ECE 153 Handout #7
Prof. Young-Han Kim Thursday, October 9, 2008

Homework Set #2
Due: Thursday, October 16, 2008

1. Read Sections 3.1–3.2, 3.4–3.6, 4.1–4.2, 4.4–4.5, 4.7, 4.9, 5.1-5.5, 5.7-5.8 in the text. Try to work on all examples.

2. *Juror's fallacy.* Suppose that $P(A|B) \geq P(A)$ and $P(A|C) \geq P(A)$. Is it always true that $P(A|B, C) \geq P(A)$? Prove or provide a counterexample.

3. Let X be a geometric random variable with pmf

$$p_X(k) = p(1-p)^{k-1}, \ \ k = 1, 2, \ldots.$$

   Find and plot the conditional pmf $p_X(k|A) = P\{X = k | X \in A\}$ if:

   (a) $A = \{X > m\}$ where m is a positive integer.

   (b) $A = \{X < m\}$.

   (c) $A = \{X \text{ is an even number}\}$.

   Comment on the shape of the conditional pmf of part (a).

4. *Negative binomial.* Suppose we observe an infinite sequence of independent coin flips with bias p (i.e., the probability of heads is p each time). Let X be the number of coin flips until observing k heads. Find the pmf of the random variable X.

5. Let $X \sim N(500, 400)$. Find

   (a) $\mathrm{P}\{480 < X < 520\}$.

   (b) $\mathrm{P}\{X < 540 | X > 460\}$.

   Hint: Use the $Q(\cdot)$ function table in the text (p. 169).

6. *Distance to the nearest star.* Let the random variable N be the number of stars in a region of space of volume V. Assume that N is a Poisson r.v. with pmf

$$p_N(n) = \frac{e^{-\rho V}(\rho V)^n}{n!}, \qquad \text{for } n = 0, 1, 2, \ldots,$$

where ρ is the "density" of stars in space. We choose an arbitrary point in space and define the random variable X to be the distance from the chosen point to the nearest star. Find the pdf of X (in terms of ρ).

7. *Random phase signal.* Let $Y(t) = \sin(\omega t + \Theta)$ be a sinusoidal signal with random phase $\Theta \sim U[-\pi, \pi]$. Find the pdf of the random variable $Y(t)$ (assume here that both t and the radial frequency ω are constant). Comment on the dependence of the pdf of $Y(t)$ on time t.

8. *Quantizer.* Let $X \sim \exp(\lambda)$, i.e., an exponential random variable with parameter λ and $Y = \lfloor X \rfloor$, i.e., $Y = k$ for $k \leq X < k+1$, $k = 0, 1, 2, \ldots$. Find the pmf of Y. Define the quantization error $Z = X - Y$. Find the pdf of Z.

9. *Gambling.* Alice enters a casino with one unit of capital. She looks at her watch to generate a uniform random variable $U \sim \text{unif}\,[0, 1]$, then bets the amount U on a fair coin flip. Her wealth is thus given by the r.v.

$$X = \begin{cases} 1 + U, & \text{with probability } 1/2, \\ 1 - U, & \text{with probability } 1/2. \end{cases}$$

Find the cdf of X.

10. Let the random variable $N(t)$ be the number of packets arriving during time $(0, t]$. Suppose $N(t)$ is Poisson with pmf

$$p_N(n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \qquad \text{for } n = 0, 1, 2, \ldots.$$

Let the random variable X be the time to get the n-th packet. Find the pdf of X.

11. *First available teller.* Consider a bank with two tellers. The service times for the tellers are independent exponentially distributed random variables $X_1 \sim \text{Exp}(\lambda_1)$ and $X_2 \sim \text{Exp}(\lambda_2)$, respectively. You arrive at the bank and find that both tellers are busy but that nobody else is waiting to be served. You are served by the first available teller once he/she becomes free. What is the probability that you are served by the first teller?