**Homework Set #6**
Due: Thursday, December 4, 2008

1. Read Sections 7.1–7.4, 9.1–9.6, 10.1–10.2, 10.4 in the text. Try to work on all examples.

2. *Symmetric random walk.* Let $X_n$ be a random walk defined as

$$X_0 = 0$$
$$X_n = \sum_{i=1}^{n} Z_i,$$

where $Z_1, Z_2, \ldots$ are i.i.d. with $\mathrm{P}(Z_1 = -1) = \mathrm{P}(Z_1 = 1) = \frac{1}{2}$.

   (a) Find $\mathrm{P}\{X_{10} = 10\}$.
   (b) Find $\mathrm{P}\{\max_{1 \le i < 20} X_i = 10 | X_{20} = 0\}$.
   (c) Find $\mathrm{P}\{X_n = k\}$.

3. *Moving average process.* Let $Y_n = \frac{1}{2}Z_{n-1} + Z_n$ for $n \ge 1$, where $Z_0, Z_1, Z_2, \ldots$ are i.i.d. $\sim N(0,1)$. Find the mean and autocorrelation function of $Y_n$.

4. *Gauss-Markov process.* Let $X_0 = 0$ and $X_n = \frac{1}{2}X_{n-1} + Z_n$ for $n \ge 1$, where $Z_1, Z_2, \ldots$ are i.i.d. $\sim N(0,1)$. Find the mean and autocorrelation function of $X_n$.

5. *Discrete-time Wiener process.* Let $Z_n$, $n \ge 0$ be a discrete time white Gaussian noise (WGN) process, i.e., $Z_1, Z_2, \ldots$ are i.i.d. $\sim N(0,1)$. Define the process $X_n$, $n \ge 1$ as $X_0 = 0$, and $X_n = X_{n-1} + Z_n$ for $n \ge 1$.

   (a) Is $X_n$ an independent increment process? Justify your answer.
   (b) Is $X_n$ a Markov process? Justify your answer.
   (c) Is $X_n$ a Gaussian process? Justify your answer.
   (d) Find the mean and autocorrelation functions of $X_n$.
   (e) Specify the first and second order pdfs of $X_n$.
   (f) Specify the joint pdf of $X_1, X_2$, and $X_3$.
   (g) Find $E(X_{15} | X_1, X_2, \ldots, X_{10})$.

6. *Random binary waveform.* In a digital communication channel the symbol "1" is represented by the fixed duration rectangular pulse

$$g(t) = \begin{cases} 1 & \text{for } 0 \le t < 1 \\ 0 & \text{otherwise,} \end{cases}$$

and the symbol "0" is represented by $-g(t)$. The data transmitted over the channel is represented by the random process

$$X(t) = \sum_{k=0}^{\infty} A_k g(t-k), \quad \text{for } t \ge 0,$$

where $A_0, A_1, \ldots$ are i.i.d random variables with

$$A_i = \begin{cases} +1 & \text{w.p. } \frac{1}{2} \\ -1 & \text{w.p. } \frac{1}{2}. \end{cases}$$

(a) Find its first and second order pmfs.

(b) Find the mean and the autocorrelation function of the process $X(t)$.

7. *Amplitude modulation.* Consider the random process $X(t) = A(t)\cos(\omega t + \Theta)$, where $A(t)$ is a zero-mean WSS process with autocorrelation function $R_A(\tau) = e^{-\frac{1}{2}|\tau|}$, $\Theta \sim$ Unif$[0, 2\pi]$, and $A(t)$ and $\Theta$ are independent. Is $X(t)$ wide sense stationary?

8. *LTI system with WSS process input.* Let $Y(t) = h(t) * X(t)$ and $Z(t) = X(t) - Y(t)$ as shown in the Figure 1.

(a) Find $S_Z(f)$.

(b) Find $E(Z^2(t))$.

Your answers should be in terms of $S_X(f)$ and the transfer function $H(f) = \mathcal{F}[h(t)]$.

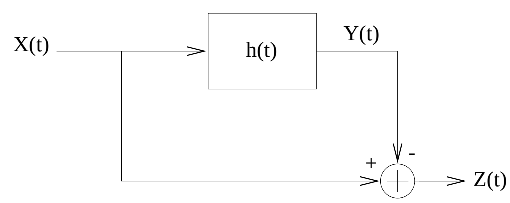


Figure 1: LTI system.

9. *Echo filtering (from Spring 2007 Final).* A signal $X(t)$ and its echo arrive at the receiver as $Y(t) = X(t) + X(t-\Delta) + Z(t)$. Here the signal $X(t)$ is a zero-mean WSS process with power spectral density $S_X(f)$ and the noise $Z(t)$ is a zero-mean WSS with power spectral density $S_Z(f) = N_0/2$, uncorrelated with $X(t)$.

(a) Find $S_Y(f)$ in terms of $S_X(f)$, $\Delta$, and $N_0$.

(b) Find the best linear filter to estimate $X(t)$ from $\{Y(s)\}_{-\infty<s<\infty}$.