

This examination is **closed book**. You may use a calculator but no reference materials. Show your work clearly and carefully, **no credit** will be given for unsupported answers.

Given formulas and constants:

Boltzman's constant $k = 1.38 \times 10^{-23}$ w/K/Hz.

If φ is $N(0, \sigma^2)$ then $\langle \exp(j\varphi) \rangle = \exp(-\sigma^2/2)$.

The far field of an aperture distribution is $E(\sin\theta_x/\lambda, \sin\theta_y/\lambda) = (\exp(-jkr)/\lambda r) \text{FT}\{E(x,y)\}$.

Effective Area = $G_{\text{MAX}} * \lambda^2/4\pi$.

$\text{FT}(\text{rect}(x)) = \text{sinc}(s) = \sin(\pi s)/\pi s$.

The magnetic field of a small electric dipole oriented in the z direction is $H_\phi(r) = j k m \sin\theta \exp(-jkr)/4\pi r$.

The reciprocity theorem is:

$$\int_V [\vec{J}_A \cdot \vec{E}_B - \vec{J}_B \cdot \vec{E}_A] dV = \int_S [\vec{E}_A \times \vec{H}_B - \vec{E}_B \times \vec{H}_A] \cdot d\vec{S}$$

Stokes' theorem is:

$$\oint \vec{A} \cdot d\vec{l} = \iint \nabla \times \vec{A} \cdot d\vec{S}$$

Questions:

1. Consider the design of a satellite radio system. You are preparing a bid on the power amplifier and the antenna, so you need a rough estimate of the total radiated power and the antenna size. The system constraints follow.

The satellite must be in a synchronous orbit (radius = 42000 km, the Earth's radius is 6400 km). The receivers are mobile and so the mobile antennas must have a broad beam and low gain because they cannot be pointed at the satellite. The satellite antenna must illuminate the whole continental USA, (about 8×10^6 km² at an average latitude of 45°N). The mobile antenna gain will be 3.0 and the mobile noise temperature will be 500°K. The frequency is 2.4 GHz. The channel bandwidth is 250 KHz, and the total bandwidth is 12.5 MHz. The user will tune only one channel at a time. A signal to noise ratio of 10 is required for adequate performance. The satellite will transmit on all 50 channels simultaneously.

- Find the total transmitted power needed.
- Find the beam solid angle of the transmitting antenna.
- Find the area of the transmitting antenna.
- What polarization will you need? Why?

2. You are designing a 2.4 GHz cell-phone system to serve Catalina Island from Los Angeles. The island subtends 30° at the headland on the Palos Verde peninsula where you will locate the base-station. You need more gain than usual to compensate for the 50 km distance to the island, so you decide to multi-beam the base-station array. You wish to have 10 beams covering the island in azimuth. The beams will overlap at the -4 dB level. The maximum elevation of the island, as viewed from the base-station, is only 2° so you can use array elements which have only 2° beam-width in elevation. The phased array elements are slotted waveguide arrays but they are phased for a fixed broadside beams so their design is straightforward. The waveguide mode is a fast wave with phase velocity = 1.5 c. The electric field polarization must be vertical.

- Find the total height of each subarray (element) (m).
- Find the slot spacing (m) in the subarrays.
- Should the slots be vertical or horizontal?
- Find the total width of the phased array (m).
- Find the gain of the whole array for the broadside beam.
- Find the number of phased elements.

- 3 (a)** Use the given formula for the far-field of a large aperture antenna to derive a formula for the aperture efficiency of the antenna when the electric field distribution over its aperture $E_{AP}(x,y)$, is known.
- (b)** Use your result to find the aperture efficiency of a lens-corrected horn excited with TE_{10} mode in rectangular waveguide. Here $E_{AP}(x,y) = \cos(\pi x/a) \Pi(x/a) \Pi(y/b)$, where $\Pi(x)$ is the unit rectangle.
- (c)** Suppose you were to adjust the horn width-to-height ratio a/b until the width of the pattern was the same in both the x -plane and the y -plane. Which would be larger a or b ? Why?
- 4.** Use the reciprocity theorem and Stokes' theorem to find the far electric field of a small current loop. Explain your procedure carefully.
- 5. (a)** Derive a formula for the power received by a radar scattered by a target with cross-sectional area σ which re-radiates isotropically. Write the expression in terms of the power transmitted P_T , the area of the radar antenna A_T , the distance and the wavelength.
- (b)** Suppose the target is a corner reflector of cross-sectional area σ . Revise the formula to calculate the received power.