This examination is **closed book**. You may use a calculator but no reference materials.
Show your work clearly and carefully, **no credit** will be given for unsupported answers.

Given formulas and constants:

Boltzman's constant k = 1.38 x 10^{-23} w/K/Hz.
If φ is N(0, σ^2) then $\langle \exp(j\varphi) \rangle = \exp(-\sigma^2/2)$.
The area of an ellipse with major and minor radii of A and B is πAB.
The far field of an aperture distribution is $E(\sin\theta_X/\lambda, \sin\theta_Y/\lambda) = (\exp(-jkr)/\lambda r)\, FT\{E(x,y)\}$.
Effective Area = $G_{MAX} * \lambda^2/4\pi$.
$FT(\prod(x)) = \mathrm{sinc}(s) = \sin(\pi s)/\pi s$.
The magnetic field of a small electric dipole oriented in the z direction is $H_\phi(r) = j\, k\, m \sin\theta \exp(-jkr)/4\pi r$.
The reciprocity theorem is:

$$\int_V [\vec{J}_A \cdot \vec{E}_B - \vec{J}_B \cdot \vec{E}_A] dV = \int_S [\vec{E}_A \times \vec{H}_B - \vec{E}_B \times \vec{H}_A] \cdot d\vec{S}$$

Stokes' theorem is:

$$\oint \vec{A} \cdot d\vec{l} = \iint \nabla \times \vec{A} \cdot d\vec{S}$$

Questions:

1 . Consider the design of a 12 Ghz satellite TV broadcast system like DirecTV. The government of Singapore has asked your firm for a bid on a system in synchronous orbit (radius = 42000 km, the Earth's radius is 6400 km) to cover Singapore only. Singapore is roughly 50 km by 30 km and lies only 1^0 north of the equator. In preliminary discussions you have convinced them that an antenna to cover just Singapore would be too large to put on the satellite and would be very difficult to keep aligned if it were built. They have negotiated a new deal that will cover Singapore and southern Malaysia up to Kuala Lumpur. That's a distance of 300 km (NW-SE) by 200 km (SE-NE) in an elliptical shape. The channel bandwidth is 10 Mhz and the total bandwidth is 1 Ghz. The receivers will be circular dishes 60 cm in diameter and will have a noise temperature of 300K. The signal to noise required for demodulation of the TV signal is 10 dB.

- **(a)** Find the flux on the ground needed per channel (w/m^2).
- **(b)** Find the transmitted power per channel and the total transmitted power needed.
- **(c)** Find the beam solid angle and the gain of the transmitting antenna.
- **(d)** Find the physical area of the transmitting antenna, assuming an aperture efficiency of 70%.
- **(e)** To illuminate an elliptical area the antenna aperture must be elliptical. What are the major and minor diameters of the antenna and what is the orientation of the major axis?

2 . You are designing a temporary 2.4 GHz cell-phone system to serve a large construction site in an uninhabited area of Northern Canada. The site is 20 km in diameter and can be covered by a base-station on a small mountain 30 km from the center of the site. There will be a lot of activity in the area and you will need 10 independent beams (in azimuth) to handle the traffic. The site is relatively flat so that a elevation angle range from 0 to $+5^0$ will be sufficient. It must be compatible with standard 2.4 Ghz handsets which are linearly polarized with the E field vertical.

- **(a)** Find the height of each subarray (element) (m) necessary to obtain the desired elevation coverage.
- **(b)** The subarrays are to be made of slotted waveguide. Must the slots be horizontal or vertical?
- **(c)** Sketch the region in $\cos(\theta)$ space that must be covered by the 10 beams. Show the visible region.
- **(d)** What is the equivalent width of each beam in $\Delta\cos(\theta)$?
- **(e)** What is the necessary width (m) of the phased array?
- **(f)** Find the maximum element spacing (m) necessary to avoid having any visible grating lobes.
- **(g)** Find the number of elements and the actual element spacing.

3 (a) Use the given formula for the far-field of a large aperture antenna to derive a formula for the aperture efficiency of the antenna when the electric field distribution over its aperture $E_{AP}(x,y)$, is known.

(b) Consider a large antenna of area A in which the feed is centered and illuminates the entire dish uniformly. However the feed blocks an area B << A. Derive a first order approximation to the effective area.

(c) The dish surface has rms errors (with respect to the ideal paraboloid) of 1 mm rms. What is the maximum frequency it can be operated at without loss of more than 1 dB in gain?

4 (a) You wish to make a "back-fire" antenna from an array of dipoles connected to an open wire transmission line. Find the element spacing and the dipole connections necessary.

(b) Suppose that you cannot *phase switch* alternate elements because your array is made with vertical monopoles fed by microstrip line. The relative dielectric constant of the microstrip $\varepsilon_R = 4$. Find the element spacing necessary for a back-fire antenna using this configuration.

5. Consider an electric dipole antenna placed near a corner where two perpendicular conducting planes intersect, parallel to the line of intersection.

(a) Find the images necessary to replace the conducting planes and sketch them on a diagram.

(b) How far from the corner should the dipole be placed to maximize the radiation?

6 (a) Use the reciprocity theorem given to find an equation for any component of the electric field of the current distribution $\mathbf{J_A}$ as a volume integral over $\mathbf{J_A}$.

(b) Use the reciprocity theorem given to find an equation for any component of the electric field of the current distribution $\mathbf{J_A}$ as a surface integral over the tangential components of $\mathbf{E_A}$ and $\mathbf{H_A}$ on that surface.