ECE 123 **Final Examination** **Winter 2008**

This exam is closed book. You may use a calculator.
Explain your work carefully, no credit will be given for unsupported answers.

Given formulas and constants:

Boltzman's constant k = 1.38 x 10^{-23} w/K/Hz.

If φ is N(0, σ^2) then $\langle\exp(j\varphi)\rangle = \exp(-\sigma^2/2)$.

The area of an ellipse with major and minor radii of A and B is πAB.

The far field of an aperture distribution is $E(\sin\theta_X/\lambda, \sin\theta_Y/\lambda) = (\exp(-jkr)/\lambda r)\, FT\{E(x,y)\}$.

$FT(\prod(x)) = \text{sinc}(s) = \sin(\pi s)/\pi s$.

$$\int_0^\pi \sin^3(\theta)d\theta = 4/3$$

$$A = G\lambda^2/4\pi$$

The reciprocity theorem is:

$$\int_V [\vec{J}_A \cdot \vec{E}_B - \vec{J}_B \cdot \vec{E}_A]dV = \int_S [\vec{E}_A \times \vec{H}_B - \vec{E}_B \times \vec{H}_A] \cdot d\vec{S}$$

Stokes' theorem is:

$$\oint \vec{A} \cdot d\vec{l} = \iint \nabla \times \vec{A} \cdot d\vec{S}$$

Pts **Questions:**

22 **1. (a)** Define the *gain* G(θ,ϕ) of a transmitting antenna in terms of its radiated flux, and show that the integral of G(θ,ϕ) over all angles = 4π.

(b) Define the *beam solid angle* Ω_B in terms of G(θ,ϕ) and derive the equation that relates Ω_B to G.

(c) Define the *effective area* A_{EFF} of a receiving antenna in terms of the power it can extract from a radiation field E(**R**).

(d) Derive the *Friis-Shelkunov* formula for the received power P_R on a link between two antennas, given the power transmitted P_T, the gain of the transmitting antenna G_T, the effective area of the receiving antenna A_R, the wavelength λ, and the distance R.

(e) Use the *Friis-Shelkunov* formula, and *reciprocity*, to prove that, for any antenna $G_{MAX} = K\, A_{EFF}$ where the constant K is the same for all antennas.

14 **2.** Consider a small dipole antenna, either an electric or magnetic dipole antenna. The radiation component of the fields will take the form of a spherical wave at large distance. E and H will have the form $E(\theta) \propto \sin(\theta)$ and $H(\theta) \propto \sin(\theta)$.

(a) Show that $G(\theta) = 1.5 \sin^2(\theta)$ regardless of which equation you start with.

(b) Find the effective area of a dipole. Explain why the effective area can be finite even though the dipole length is infinitesimal.

14 **3.** Consider a TV broadcast antenna located on a hill near Tijuana close to the border, but is designed to serve the San Diego market. It is VHF channel 7 at a frequency of 177 Mhz. Its beam width is roughly 180^0 in azimuth and 10^0 in elevation.

(a) What is the maximum gain of this antenna? How high must it be?

(b) Derive a formula for the *transmitted power*, given a requirement for the electric field strength E, at a distance R. Derive a formula for the EIRP, *effective isotropic radiated power*.

20 **4.** You are designing a linear phased-array antenna to help guide aircraft landing on a carrier. It will be mounted in a vertical position at the stern of the ship as aircraft land from that direction. It must be possible to steer the beam anywhere from broadside (zero elevation) to the +60 deg elevation. No grating lobes may appear in the visible region. The beamwidth must be 1 deg at +30 deg elevation in the middle of the steering range.

(a) Find the number of elements and the element spacing (in wavelengths) required. What will be the beamwidth at +60 deg elevation?

(b) Find the number of elements and the element spacing (in wavelengths) required if you tilt the antenna off-vertical by 30 deg? What will be the beamwidth at +60 deg elevation?

9 **5.** The magnetic field of a short dipole oriented in the z direction, with dipole moment m is given by

$$\vec{H}(\vec{r}) = H_\phi(\vec{r})\hat{\phi} = j\frac{m\sin(\theta)}{2\lambda}\frac{e^{-jkr}}{r}\hat{\phi}$$

Find the radiation resistance for length $L \ll \lambda$.

12 **6 (a)** Derive a formula for the *effective area* of a large aperture antenna when the electric field distribution over its aperture $E_{AP}(x,y)$, is known.
(b) Consider a large aperture antenna of area A in which the feed is centered and illuminates the entire dish uniformly. However the feed blocks an area $B \ll A$. Derive a first order approximation to the effective area of the antenna including the feed blockage.

8 **7.** A dish surface has errors (with respect to the ideal paraboloid) of 1 mm rms. What is the maximum frequency it can be operated at without loss of more than 1 dB in gain?

6 **8.** You would like to make a backfire antenna with an array of vertical monopoles fed by a microstrip transmission line. The relative dielectric constant of the microstrip is 2. What spacing will you need for the monopole elements?