

This problem set is due Tuesday May 19 at 11:00 AM.

**Circular Aperture Antenna:** Consider a circular aperture antenna, a parabolic reflector, which has a radius  $a = 1$  m, and operates at a wavelength of 2 cm.

(a) Assume that it is uniformly illuminated. Calculate the gain  $G(\theta)$  analytically. Plot the gain (dB) vs  $\theta$  (deg). [The 2-D Fourier transform of a unit step of radius  $a = a\lambda J_1(2\pi a\theta/\lambda)/\theta$ . The  $J_1$  function is called `besselj` in Matlab].

(b) Calculate  $G(\theta_x, \theta_y)$  using a discrete approximation with Matlab. Sample the field at  $\lambda/2$  intervals in both  $x$  and  $y$ . Sample a full  $1024 \times 1024$  square region, with the circular aperture in the center. Calculate the effective area from the aperture distribution, then get the peak gain from the effective area. Use the `fft2` function to perform a DFT on the aperture distribution. The gain is the squared magnitude of the `fft2`, but it must be properly scaled. Normalize the gain by the first point and scale it to the peak gain calculated from the effective area.

Make a three-dimensional mesh plot of  $G(\theta_x, \theta_y)$  (dB) with both  $\theta_x, \theta_y = 0$  to 5 deg. Make a two-dimensional plot of  $G(\theta_x)$  (dB) vs  $\theta_x$  (deg). Compare the 2-D plot with the theoretical plot.

(c) The first sidelobe is rather high (-17.5dB) with uniform illumination. For some purposes this must be reduced. Try tapering the aperture illumination with  $\cos(\pi r/2a)$ , and use the sampled approximation from (b) to calculate the gain.

Plot  $G(\theta_x)$  (dB) vs  $\theta_x$  (deg). Overplot  $G(\theta_x)$  for the uniform aperture for comparison. How much have you reduced the first sidelobe? What did it cost you in peak gain? In 3-dB beamwidth?

(d) Assume that the reflector surface is not perfect, so the phase in the aperture has a random component. Assume that the phase error has a Gaussian distribution with zero mean and standard deviation of 1 radian. Calculate the effect on the peak gain theoretically. Modify your code of (b) to calculate the gain including the phase error. You can use the Matlab function `randn` to generate a random variable. Find the peak gain and compare with your theoretical calculation.

Plot  $G(\theta_x)$  (dB) vs  $\theta_x$  (deg). Overplot  $G(\theta_x)$  for the uniform aperture for comparison. The total radiated power is the same, because you only changed the aperture phase. However the gain of the main beam has dropped. Where does that power go?