UCSD ECE 154C
Communications Systems III
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Solutions to ECE 154C Problem Set 1

1. **Solution:**

$$
\begin{aligned}
H_2(X) &= \sum_x p(x) log_2(1/p(x)) \\
&= \frac{4}{10} log_2(\frac{10}{4}) + \frac{3}{10} log_2(\frac{10}{3}) + \frac{3}{10} log_2(\frac{10}{3}) = 1.571.
\end{aligned}
$$

Solution is continued on scanned pages.

2. **Solution:** Refer to scanned pages.

3. a. **Solution:**

$$
\begin{aligned}
H_2(X) &= \sum_x p(x) log_2(1/p(x)) \\
&= 2 \cdot 0.1 log_2(\frac{1}{0.1}) + 0.2 log_2(\frac{1}{0.2}) + 0.6 log_2(\frac{1}{0.6}) \\
&= 1.571
\end{aligned}
$$

Solution is continued on scanned pages.

4. **Solution:**
We will use induction to prove that $H(S^n) = nH(S)$ where

$$H(S^n) = \sum_{x^n \in S^n} p(x) log(\frac{1}{p(x)}) \tag{1}$$

We denote S^n to be the set of all possible n-tuples from the alphabet S.

For $n = 1$, it is true from the definition of entropy. Now we assume

$$H(S^{n-1}) = (n-1)H(S) \tag{2}$$

Then

$$H(S^n) = \sum_{x^n \in S^n} p(x) log(\frac{1}{p(x)}) \tag{3}$$

$$= \sum_{x^{n-1} \in S^{n-1}} \sum_{x \in S} p(x^{n-1}) p(x) log(\frac{1}{p(x^{n-1} p(x))}) \tag{4}$$

$$= \sum_{x \in S} p(x) \sum_{x^{n-1} \in S^{n-1}} p(x^{n-1}) \left(log(\frac{1}{p(x^{n-1})}) + log(\frac{1}{p(x)}) \right) \tag{5}$$

$$= \sum_{x} p(x) log(1/p(x)) + \sum_{x^{n-1}} p(x^{n-1}) log(1/p(x^{n-1})) = nH(S). \tag{6}$$

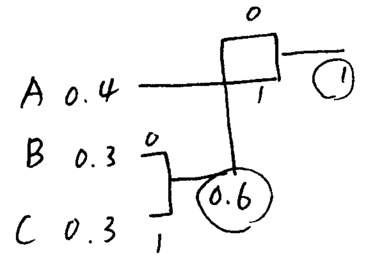
5. **Solution:**

```
% # of bits the symbols will be encoded into
bits = 0;
% generate source symbols
input_symbol = randsrc(1,1000,[0 1 2; .4 .3 .3]);
% map the source symbols into source codes
for i = 1:500
    if(input_symbol(i) == 0)
        bits = bits +3;
    elseif(input_symbol(i) == 1)
            bits = bits +3;
    else
        if(input_symbol(i+1) == 0)
            bits = bits +3;
        elseif(input_symbol(i+1) == 1)
            bits = bits +4;
        elseif(input_symbol(i+1) == 2)
            bits = bits +4;
        end
    end
end

avg = bits/1000
```

Prob 1.

$H_2(X) = 1.57$



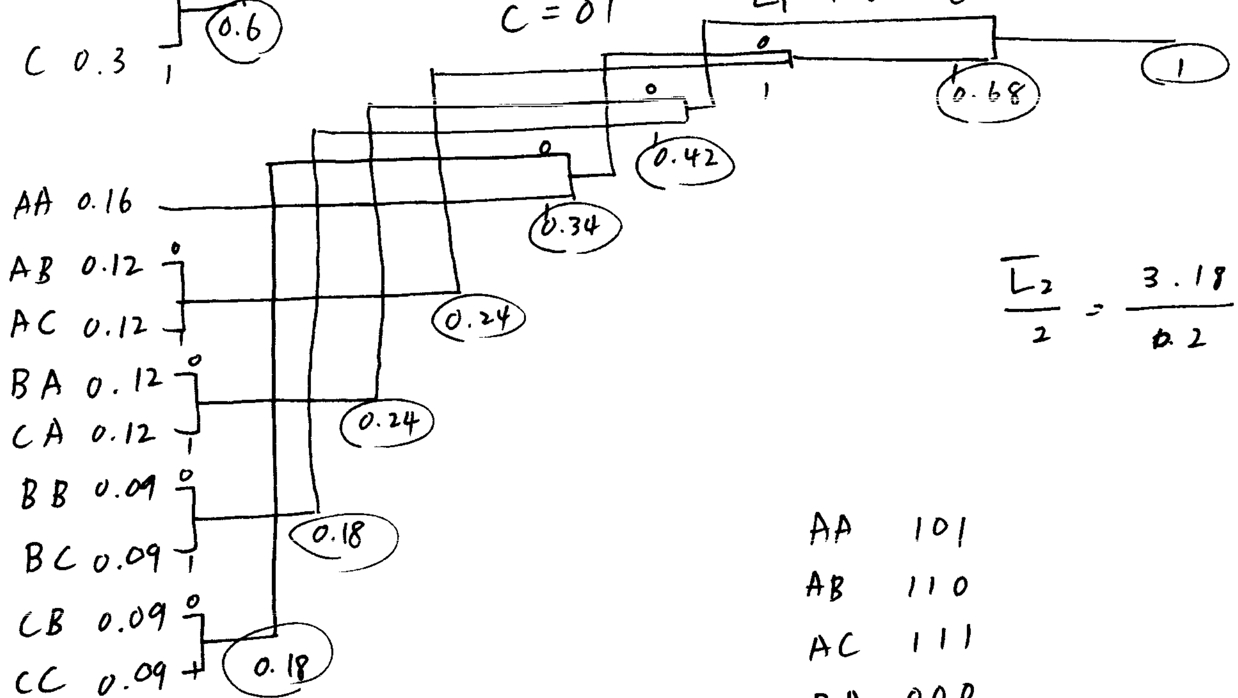
$A = 1$

$B = 00$

$C = 01$

$$\text{eff} = \frac{1.57}{1.6} = 97\%$$

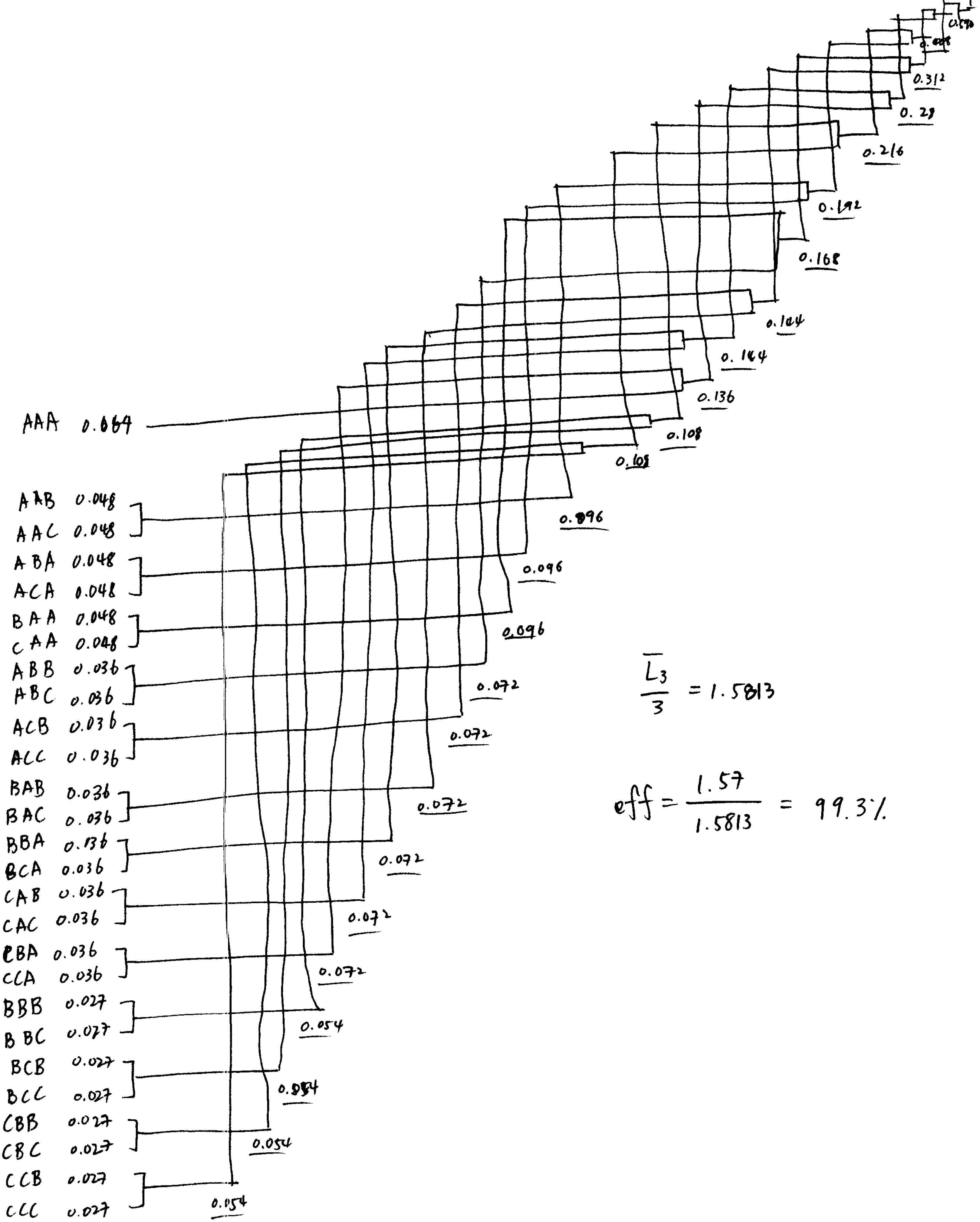
$\bar{L}_1 = 1.6$



$$\frac{\bar{L}_2}{2} = \frac{3.18}{2} = 1.59.$$

$$\text{eff} = \frac{1.57}{1.59} = 98.7\%$$

AA	101
AB	110
AC	111
BA	000
BB	010
BC	011
CA	001
CB	1000
CC	1001



$$\frac{\bar{L}_3}{3} = 1.5813$$

$$eff = \frac{1.57}{1.5813} = 99.3\%$$

Prob. 2

A	0.4	0
B	0.3	10
C	0.3	11

$\bar{L}_1 = 1.6$

$eff = \frac{1.57}{1.6} = 98.7\%$

AA	0.16	000
AB	0.12	001
AC	0.12	010
BA	0.12	011
CA	0.12	100
BB	0.09	101
BC	0.09	1100
BB	0.09	1101
CC	0.09	111

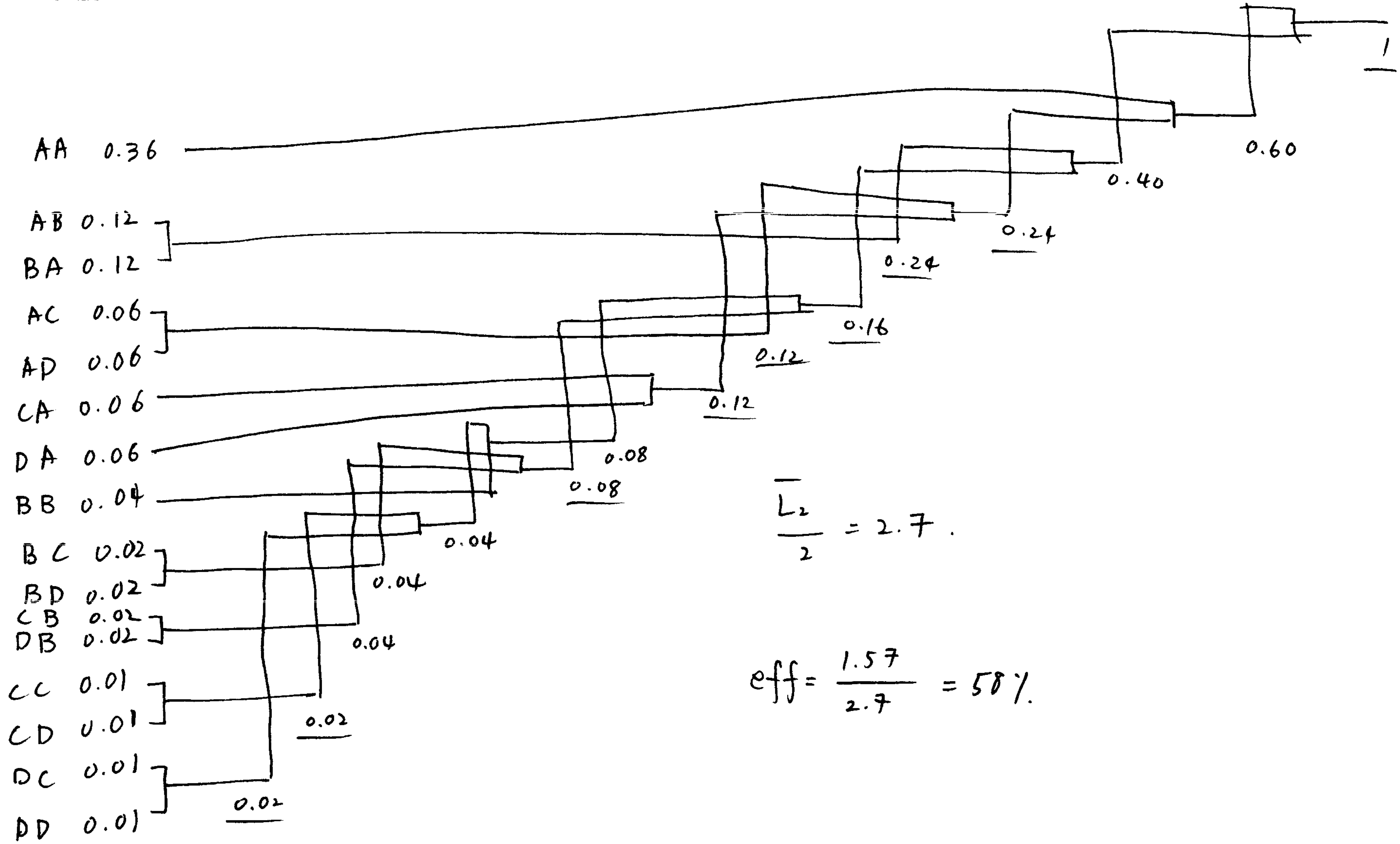
$\frac{\bar{L}_2}{2} = 1.59$

$eff = \frac{1.57}{1.59} = 98.7\%$

AAA	0.064	0000
AAB	0.048	0001
ABA	0.048	00100
BAA	0.048	00101
AAC	0.048	0011
ACA	0.048	0100
CAA	0.048	01010
ABB	0.036	01011
BAB	0.036	01100
BBA	0.036	01101
ACC	0.036	0111
CAC	0.036	10000
CCA	0.036	10001
ABC	0.036	1001
ACB	0.036	10100
BAC	0.036	10101
CAB	0.036	10110
BCA	0.036	10111
CBA	0.036	11000
BBB	0.027	11001
BCC	0.027	11010
CBC	0.027	11011
BCB	0.027	111000
BBC	0.027	111001
BCB	0.027	11101
CBB	0.027	11110
CCC	0.027	11111

$\frac{\bar{L}_3}{3} = 1.591$ $\quad eff = \frac{1.57}{1.59} = 98.7\%$

Prob 3



$$\frac{\overline{L_2}}{2} = 2.7.$$

$$\text{eff} = \frac{1.57}{2.7} = 58\%.$$