

Solutions to ECE 154C Problem Set 4

1. **Solution:**

Let $P(X = 0) = a$ and $P(X = 1) = 1 - a$ at the channel inputs.

(a) The capacity is 0.368 where $a = 0.424$.

(b) The capacity is 0.67 where $a = 0.453$.

The following is an example of the source code.

```
C = 0;

for a=0:0.00001:1
    p_x0 = a;
    p_x1 = 1-a;
    p_y0 = 0.6*a + 0.01*(1-a);
    p_y1 = 0.4*a + 0.99*(1-a);
    H_y = p_y0*log2(1/p_y0) + p_y1*log2(1/p_y1);
    p_y0gx0 = 0.6;
    p_y1gx0 = 0.4;
    p_y0gx1 = 0.01;
    p_y1gx1 = 0.99;
    H_ygx0 = p_y0gx0 * log2(1/p_y0gx0) + p_y1gx0 * log2(1/p_y1gx0);
    H_ygx1 = p_y0gx1 * log2(1/p_y0gx1) + p_y1gx1 * log2(1/p_y1gx1);
    H_ygx = a*H_ygx0 + (1-a)*H_ygx1;
    MI = H_y - H_ygx;
    if MI > C
        C = MI;
        dist = a;
    end
end
C
dist
for a=0.00001:0.00001:1
    p_x0 = a;
    p_x1 = 1-a;
    p_y0 = 0.8*a;
    p_y1 = 0.1*a + 0.1*(1-a);
```

```

p_y2 = 0.1*a + 0.9*(1-a);
H_y = p_y0*log2(1/p_y0) + p_y1*log2(1/p_y1) + p_y2*log2(1/p_y2);
p_y0gx0 = 0.8;
p_y1gx0 = 0.1;
p_y2gx0 = 0.1;
p_y0gx1 = 0.0;
p_y1gx1 = 0.1;
p_y2gx1 = 0.9;
H_ygx0 = p_y0gx0 * log2(1/p_y0gx0) +...
    p_y1gx0 * log2(1/p_y1gx0) + p_y2gx0 * log2(1/p_y2gx0);
H_ygx1 = p_y1gx1 * log2(1/p_y1gx1) +...
    p_y2gx1 * log2(1/p_y2gx1);
H_ygx = a*H_ygx0 + (1-a)*H_ygx1;
MI = H_y - H_ygx;
if MI > C
    C = MI;
    dist = a;
end
end
C
dist

```

2. Solution:

We want to calculate

$$C = \max_{P_Y(y)} I(X;Y) = \max_{P_Y(y)} (H(Y) - H(Y|X))$$

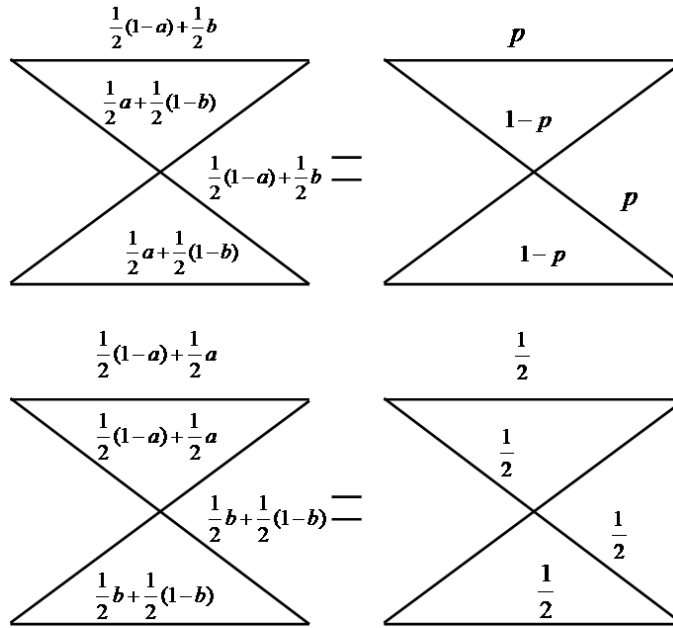
We calculate the following.

$$\begin{aligned}
 H(Y) &\leq 1/2 \log_2(2) + 1/2 \log_2(2) = \log_2(2) = 1 \\
 H(Y|X) &= \sum \sum P_X(x) P_{Y|X}(y|x) \log \frac{1}{P_{Y|X}(y|x)} \\
 &= \alpha \left((1-a) \log \frac{1}{1-a} + (1-a) \log \frac{1}{1-a} \right) + (1-\alpha) \left(a \log \frac{1}{a} + a \log \frac{1}{a} \right) \\
 &= 2\alpha(1-a) \log \frac{1}{1-a} + 2(1-\alpha) a \log \frac{1}{a} = 2\alpha - \alpha + 1 - \alpha = 1
 \end{aligned}$$

Then

$$C = H(Y) - H(Y|X) = 0$$

where $a = 1/2$.



3. Solution:

(a) We can combine the two binary channels and make it into one as the figure. Then from problem 2, the capacity is zero.

(b) We again combine the cascade of two channels into one. Then from problem 2, the capacity is zero.