

Solutions to ECE 154C Problem Set 5

1. **Solution:** Refer to scanned pages.

2. **Solution:** Refer to scanned pages.

3. **Solution:**

Let $P_c = Pr(\text{Codeword decoded wrong})$. We will not be able to correct codewords with 2,3,...,7 errors. Then

$$\begin{aligned} P_c &= Pr(\text{Codeword has 2 errors}) + \dots + Pr(\text{Codeword has 7 errors}) \\ &= \sum_{i=2}^7 \binom{7}{i} p^i (1-p)^{7-i} \\ &= 1 - \sum_{i=0}^1 \binom{7}{i} p^i (1-p)^{7-i} \\ &= 1 - (1-p)^7 - 7p(1-p)^6 \end{aligned}$$

5. **Solution:**

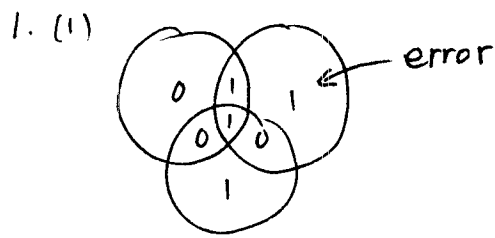
Let P_b be the probability of bit error. The upper bound of P_b can be given as the worst case where all information bits are flipped when there is a word error. The lower bound is when there is only one bit error when a word error occurs. We can write this as

$$\frac{1}{4} \leq P_b \leq P_c$$

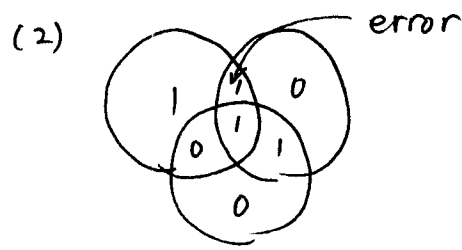
7. **Solution:**

The minimum distance of the codewords is 3. Hence it can detect at least 2 errors. We can do the following by inspecting the codewords. For 3 errors, we will fail to detect 7 patterns. For 4 errors, we will fail to detect 7 patterns. For 5 or 6 errors, we will be able to detect all patterns. For 7 errors, we fail to detect 1 pattern. Hence

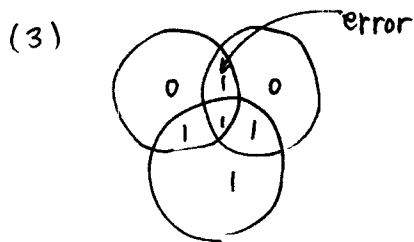
$$Pr(\text{undetected error}) = 7p^3(1-p)^4 + 7p^4(1-p)^3 + p^7$$



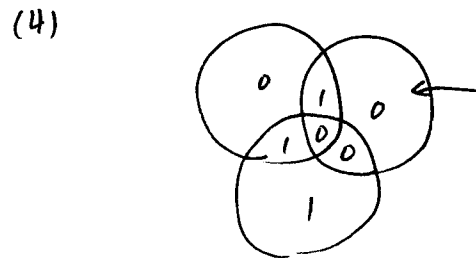
Decoded to : 1100010



Decoded to : 1010001



Decoded to : 1011010



Decoded to : 0101110

2.
$$H \approx \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

We find the column of the parity matrix that matches the syndrome of the received codewords.

(1) $\underline{S} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow 5\text{th bit}$ Decoded to : 1100010.

(2) $\underline{S} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \rightarrow 2\text{nd bit}$ Decoded to : 1010001

(3) $\underline{S} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \rightarrow 2\text{nd bit}$ Decoded to : 1011010

(4) $\underline{S} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow 5\text{th bit}$ Decoded to : 0101110