

ECE 154C
Homework #5
DUE: May 13, 2009

1. For the Hamming (7,4) code discussed in class, decode each of the following received sequences to the closest code word using the “circle method”: 1 1 0 0 1 1 0, 1 1 1 0 0 0 1, 1 1 1 1 0 1 0, and 0 1 0 1 0 1 0.
2. Repeat Problem 1, using the “syndrome decoding method”.
3. Calculate the probability of decoding to the wrong codeword if the (7,4) Hamming code is used to correct errors caused by a binary symmetric channel with bit error probability p .
4. Run a simulation that verifies the result found in problem 3. Above. That is for representative values of p (say $p=0.4, 0.3, 0.2, 0.1, 0.05, 0.01, 0.005, 0.001$) do the following a sufficient number of times to get a good estimate of the decoded word error rate. Compare your simulation results with the analytic expression found in problem 3.
 - a. Generate 4 random binary digits.
 - b. Encode them by calculating the 3 check digits.
 - c. Put the code word through a BSC with bit error probability p .
 - d. Decode the received 7-bit word.
 - e. Compare the decoded word with the transmitted word.
5. Analytically find an upper and lower bound for the error probability of the information bits. That is, after decoding to a code word, strip away the check bits and only consider the errors in the information bits. (Hint: The worst case is if all 4 information bits are in error and the best case is if only one information bit is in error.)
6. Run a simulation to verify the result found in problem 5.
7. Suppose the Hamming code is used for error detection only. For $i = 1, 2, \dots, 7$, how many error patterns with “ i ” errors will the code fail to detect?
 - a. If the code is used for error detection only on a binary symmetric channel with bit error probability p , give an analytic expression for the probability of an undetected word error as a function of p .
 - b. Verify the result found in part a by simulation for $p=0.2$.