ECE 154C
PROBLEM SET #6

DUE May 20, 2008

1. Prove that if all of the code words in a binary (n,k) code are arranged as the rows of an array, all of the columns must contain either all 0's or half zeros and half 1's.

2. Consider the binary parity check matrix with parity check matrix:

$$\begin{matrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

   a. Give the generator matrix of the code.
   b. What is the minimum Hamming distance of the code? How many errors is the code guaranteed to correct?
   c. Suppose the code is used for <u>error detection only</u>. For i = 1, 2, …, 8, how many error patterns with "i" errors will the code <u>fail to detect</u>? If the code is used for error detection only on a binary symmetric channel with bit error probability p, give an expression for the probability of undetected error as a function of p.
   d. Suppose the code is used for <u>erasure correction</u>. For i = 1, 2, …, 8, how many error patterns with "i" erasures will the code <u>correct</u>? If the code is used for erasure correction on a binary erasure channel with bit erasure probability p, give an expression for the probability that the decoder will fail to produce the correct code word as a function of p.

3. For the code described in problem 2 above, give the contents of the table that could be used to compute the parity digits from the information digits at the encoder. Describe how this same table can be used to compute the syndrome at the decoder. Use this method to compute the syndrome for the all 1 received vector (1 0 1 0 1 0 1 0).

4. Give a parity check matrix and generator matrix for a (16, 11) code with minimum Hamming distance 4 such that the last 5 columns of the parity check matrix are the unit matrix. (Hint: Start with a (15,11) Hamming single error correcting code and add an additional parity digit that forces the Hamming weight of all the code words to be even.)

5. Consider an array of 28 binary digits arranged in a rectangular array of 7 rows and 4 columns. Suppose that the number of 1's in every row and every column is even. How many distinct binary arrays of this form can be made? (Hint: Think of the last digit in every row and every column as a parity digit on that row or column.)

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