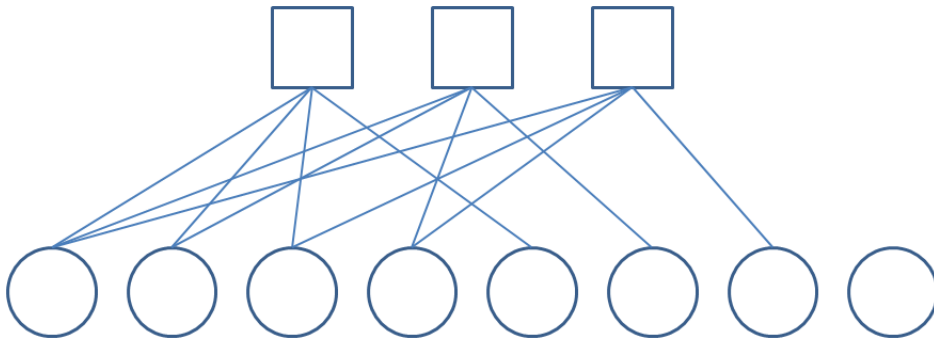


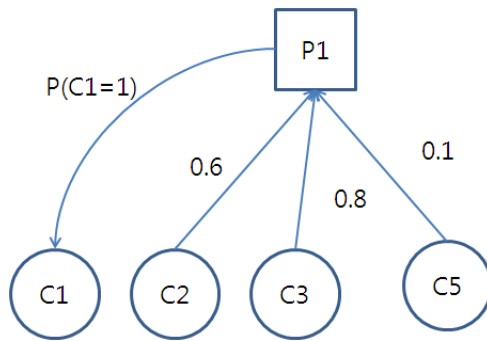
Solutions to ECE 154C Problem Set 8

1. **Solution:**

(a) The Tanner graph can be given as



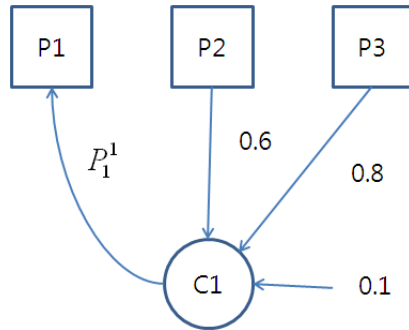
(b) We want to calculate  $P(C_1 = 1)$ .



$$\begin{aligned}
 P(C_1 = 1) &= (0.6)(0.2)(0.9) + (0.4)(0.8)(0.9) + (0.4)(0.2)(0.1) + (0.6)(0.8)(0.1) \\
 &= 0.452
 \end{aligned}$$

(c) We want to calculate  $P_1^1$ .

$$\begin{aligned}
 P_1^1 &= \frac{P_2 P_3 P_{ch}}{P_2 P_3 P_{ch} + (1 - P_2)(1 - P_3)(1 - P_{ch})} \\
 &= 0.4
 \end{aligned}$$



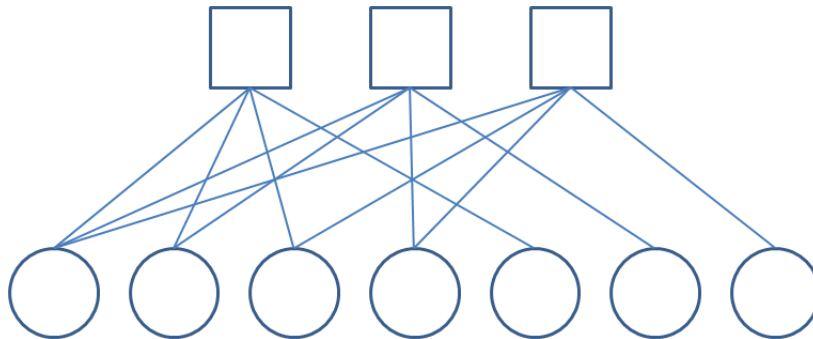
**2. Solution:**

(a) Since the code has  $d_{min} = 3$ , it is not able to correct all erasure patterns of length 3. Let  $h_1, h_2, \dots, h_7$  be the columns of the parity check matrix  $H$ . In order to correct an erasure pattern of 3, the corresponding  $h_i$ s should be linearly independent. Let's find the number of linearly independent combinations of the columns. We can pick any two arbitrary columns  $h_i$  and  $h_j$ . For the third column, we can pick any column except for the column  $h_k = h_i + h_j$ . Hence the number of independent combinations is

$$\frac{7 \times 6 \times (5 - 1)}{3!} = 28$$

The reason we divide by  $3!$  is because we do not consider the order of the columns. Therefore we are able to correct 28 patterns.

(b) The Tanner graph is as the following.



We are not able to decode the codeword if one parity node receives at least two erasures. There are 10 erasure patterns that does this. Hence we are able to decode  $35 - 10 = 25$  patterns. The uncorrectable erasure patterns are 123, 124, 127, 134, 136, 145, 234, 256, 357, 467.

(c) By adding the three parity equations we get

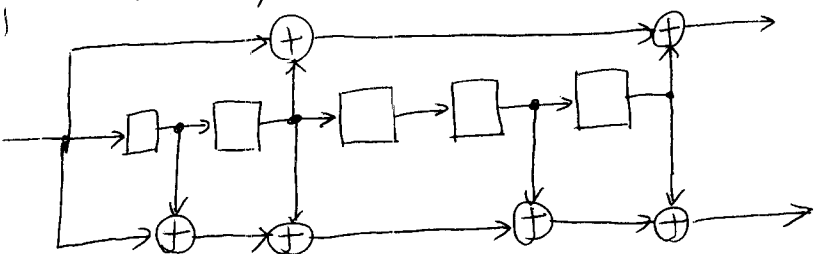
$$P_4 = C_1 \oplus C_5 \oplus C_6 \oplus C_7.$$

The minimum hamming distance of the code does not change, however we are now able to decode erasure patterns 123, 124, 134 with the new code. Therefore we can decode 28 error patterns with 3 erasures.

3. **Solution:** Refer to scanned pages.

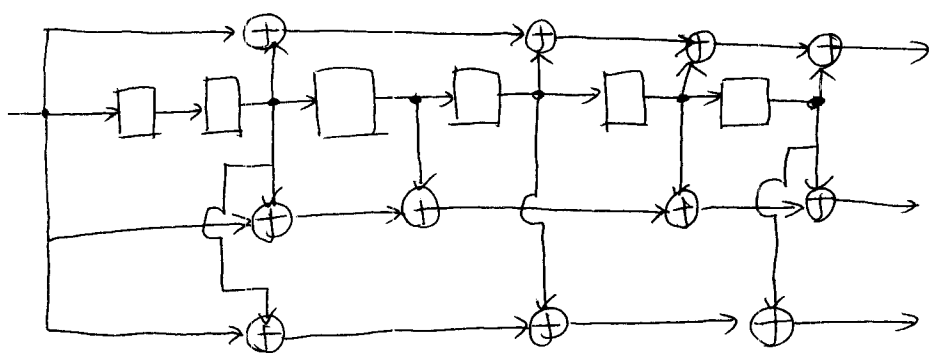
3. (a)

101001, 111011



Total states : 32

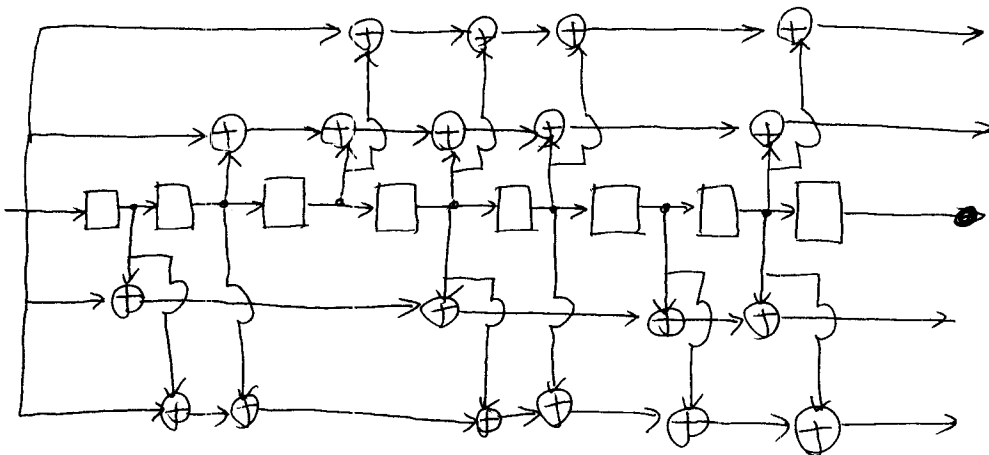
(b) 1010111, 1011011, 1010101



Total states : 64

(c)

100111010  
 101111010  
 110010110  
 111011110



Total states : 256