6.1 Electro-Optic Modulation

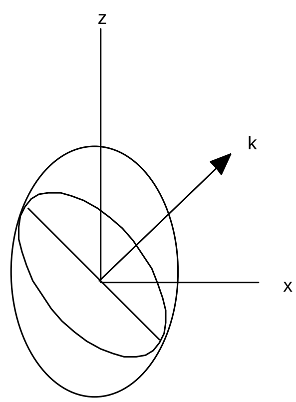
In the last two lectures we have discussed how the interaction of sound waves in a medium can deflect the light beam and also the use of the Coupled Mode Equations in the interaction of waves in a material medium. In this lecture we shall consider another material effect that causes the interaction between EM waves, namely the Electro-Optic effect.

We start with the propagation of EM waves in the material medium. For a given direction of propagation in a crystal, in general there exist two possible (linearly polarized) modes, i.e., each mode has a defined direction of polarization (or displacement vector D is determined) and experiences a particular index of refraction.

By defining the optical axis according to the crystal orientation, we can state that, using Maxwell's Equations and the condition that D.E = constant,

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \tag{1}$$

where x, y z, are defined as the principal axes where the electric field vector is parallel to D. Geometrically speaking, Equation 1 can be represented by an ellipsoid,



Pockel effect is another common name for electro-optic effect, it describes the index change in the presence of the electric field. When the index change is linearly proportional to the E field, we have the linear electro-optic effect. It is observed that a crystal with inversion symmetry (f(x) = f(-x)) does not possess linear E-O effect.

$$\Delta n_1 = s\,E$$

$$\Delta n_2 = s\,(-E)$$

Because of the inversion symmetry, $\Delta n_1 = \Delta n_2$, or s= 0.

Instead of working directly with Δn, we could use the index ellipsoid and say that, due to the presence of the E-field vector, Eq. 1 is modified to:

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 zx + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \qquad (2)$$

where x, y, z, are parallel to the principal axes of the crystal and at E = 0,

$$\left(\frac{1}{n^2}\right)_i = \frac{1}{n_i^{\,2}} \qquad \text{(for } i = 1,2,3\text{)}$$
$$\left(\frac{1}{n^2}\right)_j = 0 \qquad \text{(for } j = 4,5,6\text{)}$$

The linear change in $(1/n^2)_i$ due to the nonzero E can be defined in term of a tensor $\{r_{ij}\}$, where

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^{3} r_{ij}E_j \qquad (3)$$

and $\{r_{ij}\}$ is a 3 x 6 tensor,

$$\{r\} = \begin{pmatrix} r_{11} & \cdots & r_{13} \\ \vdots & \ddots & \vdots \\ r_{61} & \cdots & r_{63} \end{pmatrix} \qquad (4)$$

For example,

$$\Delta\left(\frac{1}{n^2}\right)_6 = r_{61}E_1 + r_{62}E_2 + r_{63}E_3 \qquad (5)$$

Please see Yariv's Optical Electronics for different examples of {r}.

For the tetragonal ($4\underline{2}m$) KH_2PO_4 (KDP), the nonzero elements $r_{41} = r_{52}$, r_{63} are as follows:

	$\lambda = 0.546\ \mu m$	$\lambda = 0.633\ \mu m$
r_{41}	8.77	8
r_{63}	10.3	11
n_o	1.5079	1.502
n_e	1.4683	1.462

For this material, the crystal is uniaxial crystal so two of the indices are identical (at E=0), say $n_x = n_y$. Eq. 2 becomes,

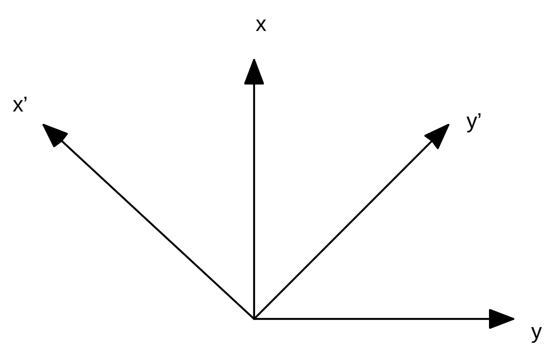
$$\left(\frac{1}{n_o^2}\right)x^2 + \left(\frac{1}{n_o^2}\right)y^2 + \left(\frac{1}{n_e^2}\right)z^2 + 2r_{41}E_x yz + 2\,r_{41}E_y zx + 2r_{63}E_z xy = 1 \qquad (6)$$

This can be interpreted by having new principal axes right after the E field is turned on! Take the case of $E = E_z$, the index ellipsoid becomes,

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\,r_{63}E_z xy = 1 \qquad (7)$$

We can define a new x'- y' axes which is obtained by a 45° CCW rotation of the x-y axes:

$$x = x'\cos 45^o + y'\sin 45^o$$
$$y = -x'\sin 45^o + y'\cos 45^o$$



Then Eq. 7 becomes:

$$\frac{\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2}{n_o^2} + \frac{\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)\left(\frac{-x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 1$$

or,

$$\frac{x'^2 + y'^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\, r_{63} E_z \left(\frac{y'^2}{2} - \frac{x'^2}{2} \right) = 1 \qquad (8)$$

or,

$$x'^2 \left(\frac{1}{n_o^2} - r_{63} E_z \right) + y'^2 \left(\frac{1}{n_o^2} + r_{63} E_z \right) + \frac{z^2}{n_e^2} = 1 \qquad (9)$$

With this new co-ordinate system, the new x-index is:

$$n_x'^2 = n_o^2 \left(1 - n_o^2 r_{63} E_z \right)^{-1}$$

For the case that,

$$r_{63} E_z << \frac{1}{n_o^2}$$

we have,

$$n_x' \approx n_o + \frac{1}{2} r_{63} E_z\, n_o^3 + \ldots \qquad (10)$$

Similarly, we get,

$$n_y' \approx n_o - \frac{1}{2} r_{63} E_z\, n_o^3 + \ldots \qquad (11)$$

From Eqs. 10, 12 we can see the push-pull relationships between the phase components along x', y' directions.

6.2 General Approach to Index Ellipsoid Equation

If we write out a 3 x 3 matrix S as follows:

$$S_{11}=\left(\frac{1}{n^2}\right)_1,\ S_{22}=\left(\frac{1}{n^2}\right)_2,\ S_{33}=\left(\frac{1}{n^2}\right)_3$$
$$S_{32}\ = S_{23}\ \left(\frac{1}{n^2}\right)_4, \text{ etc.}$$

Eq. 2 can be written in a more compact manner,

$$\sum_{i,j}^{3} S_{ij}x_i x_j = 1 \qquad (12)$$

Or in the vector form,

$$\begin{pmatrix} x_1\ x_2\ x_3 \end{pmatrix}\begin{pmatrix} S \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1 \qquad (13)$$

We can use vector algebra to simplify Eq. 12. For any point x, let's define a vector N, such that N_i is,

$$N_i = \sum_{j}^{3} S_{ij}x_j$$

Eq. 12 means N. x =1, or geometrically N is normal to the ellipsoid at x. Since for principal axes system, the principal axes are normal to the surface, so N must also be parallel to the principal axes, or we can solve for both by requiring:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = s\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

This is equivalent to solving,

$$\begin{pmatrix} (S_{11}-s) & S_{12} & S_{13} \\ S_{21} & (S_{22}-s) & S_{232} \\ S_{31} & S_{32} & (S_{22}-s) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad (14)$$

The determinant of Eq. 14 must vanish, which gives the characteristic equation for determining the different s's. Once s_k is determined, we can determine the radius of the corresponding principal axis:

$$|x|_k = \frac{1}{\sqrt{s_k}}$$

For instance we can re-do the example above, the S matrix is

$$\begin{pmatrix} \frac{1}{n_o^2} & r_{63}E_z & 0 \\ r_{63}E_z & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{pmatrix}$$

So we set,

$$\det \begin{pmatrix} \frac{1}{n_o^2} - s & r_{63}E_z & 0 \\ r_{63}E_z & \frac{1}{n_o^2} - s & 0 \\ 0 & 0 & \frac{1}{n_e^2} - s \end{pmatrix} = 0$$

and get,

$$\left(\frac{1}{n_e^2} - s\right)\left[\left(\frac{1}{n_o^2} - s\right)^2 - \left(r_{63}E_z\right)^2\right] = 0$$

from which we obtain,

$$s_1 = \frac{1}{n_e^2}$$
$$s_2 = \frac{1}{n_o^2} + r_{63}E_z$$
$$s_3 = \frac{1}{n_o^2} - r_{63}E_z$$

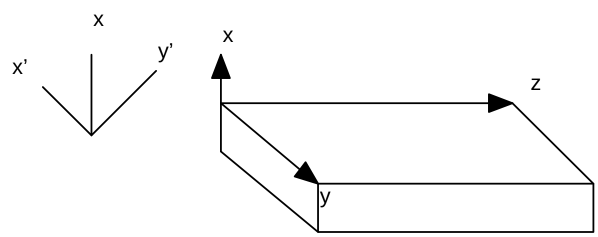
The principal axes can be obtained by direct substitution of s_k's back to Eq. 14.

6.3 Electro-optic Retardation

We proceed by using KDP as an example, take $E=E_z$ again, and set z = 0, from Eq. 9, we get,

$$x'^2\left(\frac{1}{n_o^2} - r_{63}E_z\right) + y'^2\left(\frac{1}{n_o^2} + r_{63}E_z\right) = 1$$

For a given cut of the crystal, the x-, y- axes are assumed to be pre-determined. With the incident light polarized along the x direction, we can resolve the electric field into x' and y' components,



$$e_{x'} = Ae^{i\left(\omega t - \frac{\omega}{c} n_{x'} z\right)} = Ae^{i\left(\omega t - \frac{\omega}{c}\left(n_o + \frac{n_o^3}{2} r_{63}E_z\right)z\right)} \quad (15)$$

$$e_{y'} = Ae^{i\left(\omega t - \frac{\omega}{c} n_{y'} z\right)} = Ae^{i\left(\omega t - \frac{\omega}{c}\left(n_o - \frac{n_o^3}{2} r_{63}E_z\right)z\right)} \quad (16)$$

These two components will thus travel at phase velocities (since they have different index) through the medium and will end up with different phases at the output plane z = L. The phase difference between the two components is:

$$\Gamma = \phi_{x'} - \phi_{y'} = \frac{\omega}{c} n_o^3 r_{63} E_z L = \frac{\omega}{c} n_o^3 r_{63} V \quad (17)$$

where V is the voltage applied across L. (Therefore the longer L is, the higher the voltage needed for a given phase change!) If Γ equals $\pi/2$, the two components at the output will combine to form a circularly polarized light. At $\Gamma = \pi$, we get the linearly polarized light again except it is 90^o from the original beam (y-polarized).

It is customary to define a half wave voltage, V_π, which is the voltage required for a π phase shift. From Eq. 17, we have,

$$V_\pi = \frac{\pi c}{\omega\, n_o^3 r_{63}} \quad (18)$$

For ADP material, for instance, r_{63} is 8.56 x 10^{-12} m/V at λ =0.5 μm, and n_o is 1.5, this gives a V_π of ~ 10 kV!