Acousto-Optic Modulation

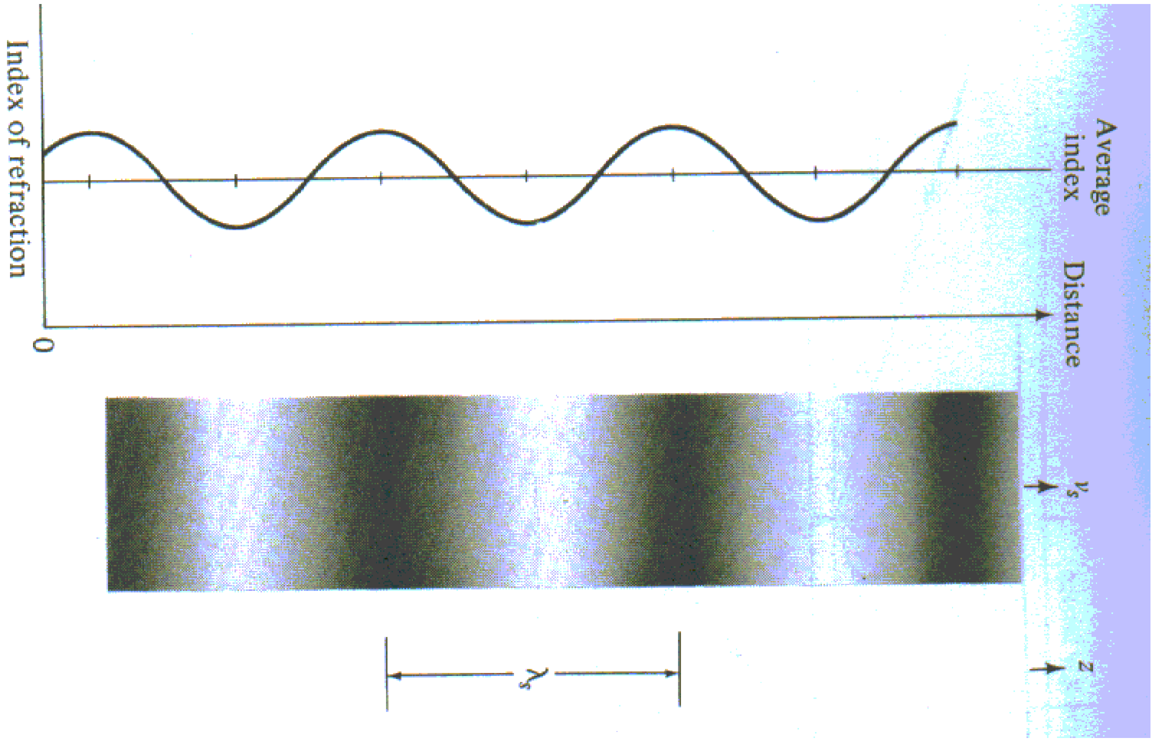
8.1 Interaction of light and sound - An Introduction

Reference: see A. Yariv's Optical Electronics

The acousto-optic modulation can be explained as follows. A sound wave causes a sinusoidal perturbation in the density of the material, which shows up as a strain wave traveling at the sound velocity, V_s, across the materials. The change of density corresponds to a change of index, Δn, which is responsible for the modulation of a propagating electromagnetic wave.

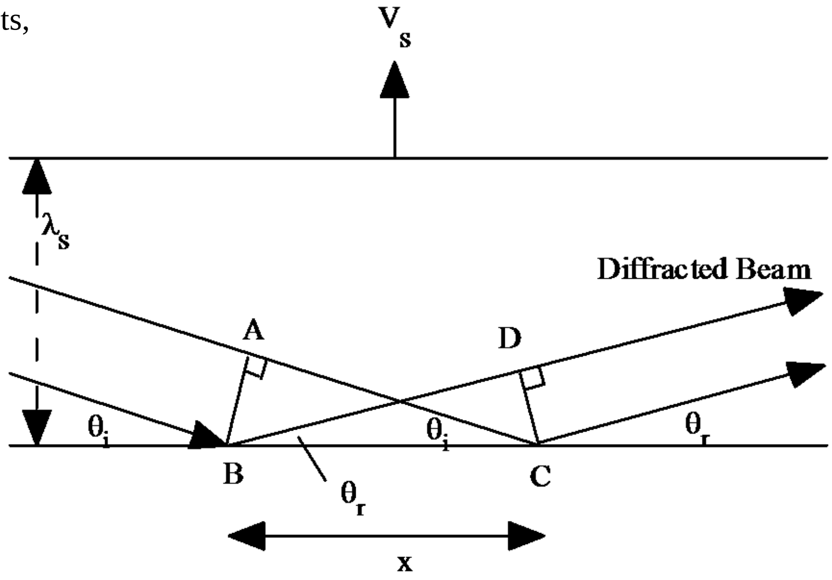
$$\Delta n(r,t) = \Delta n \cos(\omega_s t - k_s z) \qquad (1)$$

Since the "waves" in an optical beam move very much faster than that of the sound wave, the $\Delta n(z,t)$ created by the sound wave appears stationary over many cycles of the optical beam. So the modulation effect of the sound wave resembles that of a fix grating (which is actually moving).



We can characterize the sound waves as series of partially reflecting mirrors, separated by the sound wavelength λ_s, that are moving at the velocity V_s. For the diagram below, if

B, C represents two equal reflection points at the crystal, i.e. the index change is the same at both points,



For strong diffraction to occur in a given direction, all these points in the mirror contribute in phase to the diffraction along this direction. Therefore, we need the path difference between two beams equals to an integral multiple of the wavelength in the medium, or since the path difference is AC-BD,

$$AC - BD = m\frac{\lambda}{n} \tag{2}$$

where m is an integer, n is the medium index. From the diagram, we also have,

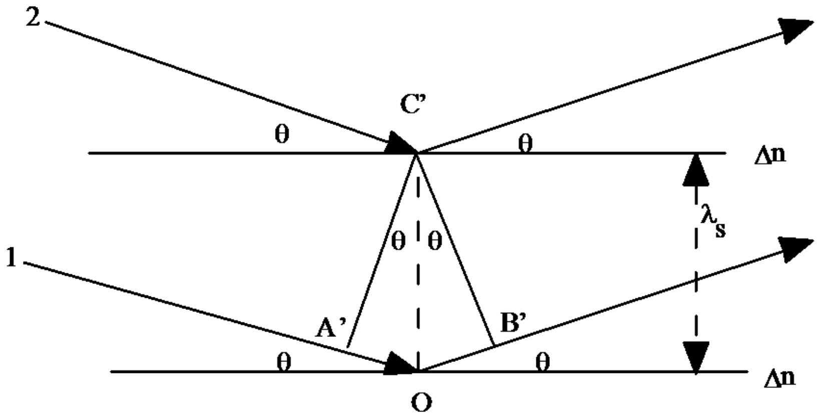
$$AC = x\cos\theta_i \ , BD = x\cos\theta_r \tag{3}$$

Therefore, Eq. 3 becomes

$$x(\cos\theta_i - \cos\theta_r) = m\frac{\lambda}{n} \tag{4}$$

Since x is arbitrary, the only way this works for all points in a given reflector is that we have the trivial case of m = 0, $\theta_i = \theta_r$ (the law of reflection).

In addition, we also require that the diffraction from any two acoustic phase fronts add up in phase along the direction of the reflected beam:



(Note that the incident beam has a frequency ω, the diffracted beam has a frequency ($\omega + \omega_s$, where ω_s is the frequency of the moving sound wavefront at the velocity V_s)

Here A'O + B'O is the additional distance traveled by beam 1. In order to obtain a constructive interference between the two beams,

$$A'O + B'O = m' \frac{\lambda}{n} \tag{5}$$

From the diagram we can see

$$A'O = \lambda_s \sin\theta$$

so Eq. 5 becomes

$$2\lambda_s \sin\theta = m' \frac{\lambda}{n} \tag{6}$$

For the first order diffraction, m' =1, $\theta = \theta_1$, Eq. 6 is known as the Bragg Diffraction condition. (The second order and so on can occur for thin samples).

Example: For sound wave with $V_s = 3 \times 10^5$ cm/s, a frequency (ν_s) of 500 MHz, and an optical wavelength of 0.5 μm (λ_{opt}/n). This corresponds to a grating period (spacing) of $\lambda_s = V_s/\nu_s = 6$ μm. We can use Eq. 6 to find that $\sin(\theta) = \lambda_{opt}/(2n\lambda_s) \sim 0.5/12$, so $\theta \sim 3.5^o$ is indeed very small.

8.2. Particle Picture of Bragg Diffraction of light

Photon, having the nature of wave, can be represented by a propagation vector k and so its phase at a given point x takes the exp i(ωt + k.x) dependence. In this formulation, the incident photon has momentum ∇k_i (as a particle), and energy $\nabla \omega_i$; the diffracted photon has momentum ∇k_d, and energy $\nabla \omega_d$. The sound 'wave' has momentum ∇k_s, energy $\nabla \omega_s$. The interaction of the three particles involves integration like this over the medium:

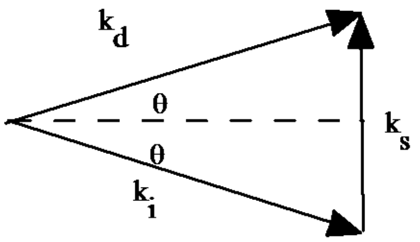
$$\int d^3r\, e^{-i(\omega_d t + k_d x)}\, \Delta n(z,t)\, e^{i(\omega_i t + k_i x)} \qquad (7)$$

To get a non-zero integration we need to have a zero exponent, or

$$k_d = k_i \pm k_s \qquad (8)$$

$$\omega_d = \omega_i \pm \omega_s \qquad (9)$$

If we remember that both ω_i and ω_d are much larger than ω_s, this means that $\omega_i \sim \omega_d$ in value and the magnitude of k_i and k_d are nearly the same. From Eq. 8, we have:



This figure gives $k_s = 2\, k \sin\theta$ which is identical to Eq. 6 with m' =1.

For the long interaction length case, i.e., the medium is large in extent, only the m=1 solution survives.

8.3. Coupled Wave (mode) analysis of A-O modulation

Coupled mode analysis is a very powerful technique to describe the interaction of electromagnetic waves in the presence of a material medium. Recall that one of the assumptions of basic E&M is that the electric fields do not interact in vacuum. However, in material medium, dipoles are induced and they can interact with waves. From E&M, the displacement vector is

$$D = \varepsilon_o E + P = \varepsilon E \tag{10}$$

where P is the polarization of the medium. For simplicity, we assume the average index of the medium is close to that of vacuum:

$$\begin{aligned} P &= (\varepsilon - \varepsilon_o) E = \varepsilon_o \left(\frac{\varepsilon}{\varepsilon_o} - 1\right) E = \varepsilon_o (n^2 - 1) E \\ &= \varepsilon_o (n + 1) \Delta n\, E \approx \varepsilon_o (2n) \Delta n\, E \\ &\approx 2\sqrt{\varepsilon \varepsilon_o} \Delta n(r,t)\, E(r,t) \end{aligned} \tag{11}$$

In these equations E is the sum of the two electric fields, E_i and E_d, and $\Delta n(r,t)$ is as before,

$$\Delta n(r,t) = \Delta n \cos(\omega_s t - k_s z) \tag{1}$$

P can be regarded as the agent that facilitates the power exchange between E_i and E_d each of which have to satisfy the wave equation separately:

$$\nabla^2 E_{i\,or\,d}(r,t) = \mu\, \varepsilon \frac{\partial^2 E_{i\,or\,d}}{\partial t^2} + \mu \frac{\partial^2 P_{\mathrm{i\,or\,d}}}{\partial t^2} \tag{12}$$

where P_i is the component of P along the direction of E_i. Take $E_i(r,t)$ to be:

$$E_i(r,t) = \frac{1}{2} E_i(r_i) e^{i(\omega_i t - k_i . r_i)} + \text{c.c.} \tag{13}$$

where c.c. stands for complex conjugate.

$$\nabla^2 E_i(r,t) = -\frac{1}{2}\left(k_i^{\,2} E_i + 2i\, k_i \frac{\partial E_i}{\partial r_i} + \nabla^2 E_i(r)\right) e^{i(\omega_i t - k_i . r_i)} + \text{c.c.} \tag{14}$$

The third term inside the bracket is slowly varying and can be ignored, and in the absence of the sound wave perturbation,

$$k_i^{\,2} = \omega_i^{\,2}\mu\varepsilon_o \qquad (15)$$

Substitute Eq. 14 into Eq. 12, and note that the first term cancels the first term on the right side by virtue of Eq. 15, by collecting the remaining terms we get,

$$\mathrm{k_i}\frac{\partial E_\mathrm{i}}{\partial r_\mathrm{i}} = \mathrm{i}\mu\frac{\partial^2 P_i}{\partial t^2}\mathrm{e}^{-\mathrm{i}(\omega_\mathrm{i}t - \mathrm{k_i.r_i})} \qquad (16)$$

From Eqs. 11, 1, and 13, we have,

$$\mathrm{P} = 2\sqrt{\varepsilon\varepsilon_\mathrm{o}}\,\Delta\mathrm{n(r,t)}\ (\mathrm{E_i(r,t)} + \mathrm{E_d(r,t)}) \qquad (17)$$

When Eq. 17 is substituted into Eq. 16 and after grouping the ωt factor of and Δn, E_i and E_d, (see Eq. 13), we can collect terms that nearly cancel in the ωt part of exponents. For instance, the relevant terms in P that meet the condition $\omega_i = \omega_s + \omega_d$ are

$$\mathrm{P_i} = \frac{1}{2}\sqrt{\varepsilon\varepsilon_\mathrm{o}}\,\Delta\mathrm{n}\ \mathrm{E_d}\left(\mathrm{e}^{\mathrm{i}(\omega_\mathrm{s} + \omega_\mathrm{d})t - \mathrm{i}(\mathrm{k_s} + \mathrm{k_d}).\mathrm{r}}\right) + \mathrm{c.c.} \qquad (18)$$

(now there is a factor of ½ from Δn, another ½ from E, together they give a factor of ¼) which leads to,

$$\frac{\mathrm{dE_i}}{\mathrm{dr_i}} = -\,\mathrm{i}\,\eta_\mathrm{i}\,\mathrm{E_d}\,\mathrm{e}^{\,\mathrm{i}(\mathrm{k_i} - \mathrm{k_s} - \mathrm{k_d}).\mathrm{r}} \qquad (19)$$

where

$$\eta_\mathrm{i} = \frac{1}{2}\omega_\mathrm{i}\sqrt{\mu\varepsilon_\mathrm{o}}\,\Delta\mathrm{n}$$

Similarly, starting with the wave equation for E_d, we get,

$$\frac{\mathrm{dE_d}}{\mathrm{dr_d}} = -\,\mathrm{i}\,\eta_\mathrm{d}\,\mathrm{E_i}\,\mathrm{e}^{-\,\mathrm{i}(\mathrm{k_i} - \mathrm{k_s} - \mathrm{k_d}).\mathrm{r}} \qquad (20)$$

with

$$\eta_d = \frac{1}{2}\,\omega_d\sqrt{\mu\varepsilon_o}\,\Delta n$$

Eq. 19 and Eq. 20 are the coupled mode equations for this acousto-optic system.