8.3 AO Modulation (continued)

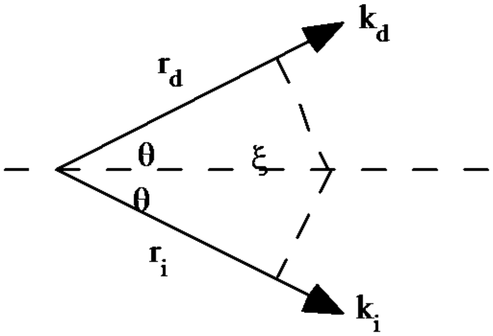
From the discussion of the Coupled Mode Equations, we note that the critical physical quantity responsible for the mode coupling is the index change (the η term in Eq. 19 and Eq. 20). As mentioned last time, $\omega_i \sim \omega_d$, so $\eta_i \sim \eta_d = \eta$. When the Bragg condition is met, we have,

$$\frac{dE_i}{dr_i} = -i\eta E_d \tag{1}$$

and

$$\frac{dE_d}{dr_d} = -i\eta E_i \tag{2}$$

where r_i and r_d are vectors along the beams (i.e. parallel to k_i, k_d respectively). We have the following simple picture,



This way we can transform the co-ordinates from r_i and r_d to ξ and θ as $|r_i| = |r_d| = \xi \cos\theta$ (i.e. they have the same projection on the ξ axis). Eq. 1 then becomes,

$$\frac{dE_i}{d\xi} = \frac{dE_i}{dr_i}\cos\theta = -i\eta E_d \cos\theta \tag{3}$$

Similarly,

$$\frac{dE_d}{d\xi} = -i\eta E_i \cos\theta \tag{4}$$

The coupled equations (3) and (4) can be solved together (by taking the 2nd derivative w.r.t ξ),

$$E_i(\xi) = E_i(0)\cos(\eta\xi\cos(\theta)) - iE_d(0)\sin(\eta\xi\cos(\theta)) \tag{5}$$

By examining the above figure, the origin of r_i, r_d, ξ coincide, and

$$E_i(\xi) = E_i(0)\cos(\eta r_i) - iE_d(0)\sin(\eta r_i) = E_i(r_i) \tag{6}$$

Similarly,

$$E_d(r_d) = E_d(0)\cos(\eta r_d) - iE_i(0)\sin(\eta r_d) \tag{7}$$

Note that the two beams interact continuously along the "vibrating" medium.

For the special case that only the incident beam exists at the origin, $E_d(0) = 0$, we have

$$E_i(r_i) = E_i(0)\cos(\eta r_i)$$
$$E_d(r_d) = -iE_i(0)\sin(\eta r_d)$$

In this case we see that a complete transfer of power from the incident beam to the diffracted beam occurs wherever

$$\eta r_i = \eta r_d = \frac{\pi}{2}$$

Thus, along the way,

$$\frac{I_{\text{diffracted}}}{I_{\text{incident}}(0)} = \frac{E^2_{\text{diffracted}}}{E_i^2(0)} = \sin^2(\eta r_d) = \sin^2\left(\frac{\omega l}{2c}\Delta n\right) \qquad (8)$$

As will be shown later, the photoelastic index change is given in terms of the strain s and the photoelastic constant p,

$$\Delta n = -\frac{n^3 ps}{2}$$
$$s = \sqrt{\frac{2I_{\text{acoustic}}}{\rho V_s^3}}$$

where I_{acoustic} is the acoustic intensity, ρ is the mass density and V_s is the sound velocity in the medium. Substituting this in Eq. 8, we get

$$\frac{I_{\text{diffracted}}}{I_{\text{incident}}(0)} = \sin^2\left(\frac{\pi l}{\sqrt{2}\lambda}\sqrt{MI_{\text{acoustic}}}\right) \qquad (9)$$

where M is a material constant given by

$$M = \frac{n^6 p^2}{\rho V_s^3}$$

Numerical example:

Given a medium (water) with $n = 1.33$, $p = 0.31$, $V_s = 1.5 \times 10^3$ m/s, $\rho = 1000$ kg/m^3, at the laser wavelength of 632.8 nm, we expect the ratio to be:

$$\sin^2 (1.4\, l \sqrt{I_{acoustic}})$$

where the length l is measured in μm.

The above is for the case where the Bragg condition is met. For the case when there is a small derivation from the Bragg condition, the diffraction efficiency becomes:

$$\frac{I_{diffracted}}{I_{incident}(0)} = \frac{E^2{}_{diffracted}}{E_i^2(0)} = \left(\frac{cl}{sl}\right)^2 \sin^2 (sl) \qquad (8a)$$

where c is the overlap of the permittivity change with the electric field intensity profile (across the transverse direction x, note that the electric field profile is a property of the waveguide):

$$c = \frac{\omega}{4} \int \Delta\varepsilon \left|E^2\right| dx$$

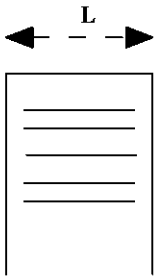
and sl is given by

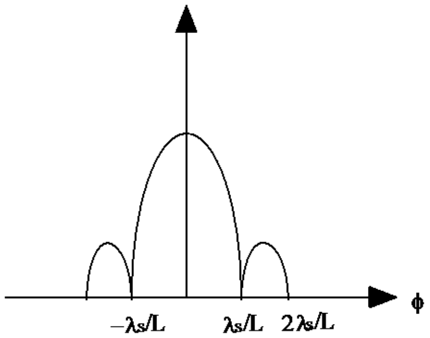
$$sl = \sqrt{(cl)^2 + \left(\frac{l\Delta k_z}{2}\right)^2}$$

where Δk_z denotes the difference of the z-component of k_i and k_d.

8.4 The Bulk-Wave A-O Beam deflector

The above analysis is based upon the assumption that the acoustic wave is an ideal plane wave, that is, it has an infinite spatial extent transverse to the acoustic propagation direction. In practice, the device has a finite width, say L. For a finite aperture beam, the diffraction pattern has an angular spread, $\delta\phi = 2\lambda_s/L$, where $\delta\phi$ represents the angular spread between the first set of nulls in the Fourier spectrum:

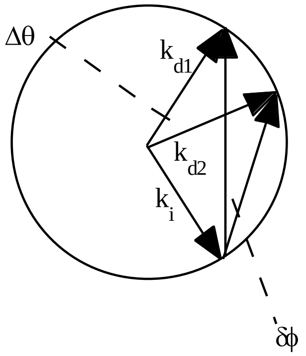


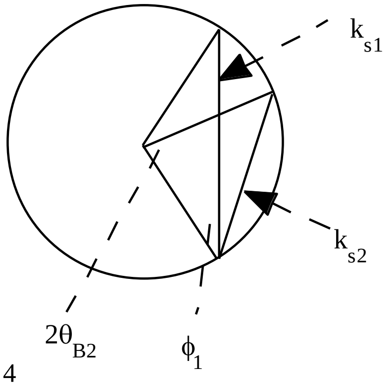


A more conservative measure of the angular spread is to take one half of the above value or

$$\delta\phi_{max} \cong \frac{\lambda_s}{L} \tag{10}$$

This finite spread means it is possible for Bragg condition to be satisfied over a range of frequencies. For example, let's say the Bragg condition is satisfied by two acoustic frequencies f_1 and f_2, which correspond to the plane wave components k_{s1} and k_{s2} respectively. Provided that angular spread between k_{s1} and k_{s2} is much less than λ_s/L, the amplitude of the acoustic plane waves associated with these components are comparable and thus the respective deflected optical waves have nominally equal amplitudes. Consider the first order (m=1) diffraction:





note that from these figures:

$$2\theta_{Bi} + 2\phi_i = \pi$$

so that

$$\Delta\theta = 2\theta_{B1} - 2\theta_{B2} = 2(\phi_2 - \phi_1) = 2\delta\phi \qquad (11)$$

From the Bragg condition:

$$\sin\theta_{Bi} = \frac{\pi}{\lambda_{si}k} = \frac{\lambda}{2\lambda_{si}}$$

We have previously shown that θ is very small, so

$$2\theta_{Bi} \approx \frac{\lambda}{\lambda_{si}} = \frac{f_i}{V_s}\lambda \qquad (12)$$

and the angular spread of the diffracted beam is:

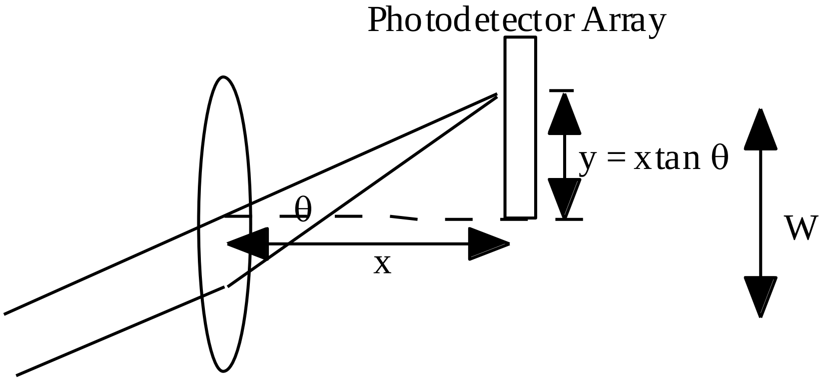
$$\Delta\theta = \frac{f_1 - f_2}{V_s}\lambda = \frac{\Delta f}{V_s}\lambda = 2\delta\phi$$

From Eq. 10, we see that the maximum bandwidth that satisfy the Bragg condition is

$$\Delta f_{RF} = 2\frac{V_s}{\lambda}\frac{\lambda_s}{L} \qquad (13)$$

8.5 Acousto-optic spectrum analyzer

The diffracted beam is focussed by an optical element (a lens) to a detector array. Let's examine the following detection scheme:



The minimum spread of the beam is δθ ~ λ/W, where W is optical beam width, or beam width of a coherent beam. In general W is proportional to the time τ during which the acoustic grating appears stationary (or transit time of the beam), so W ~ V_s τ.
Since θ is small, therefore:

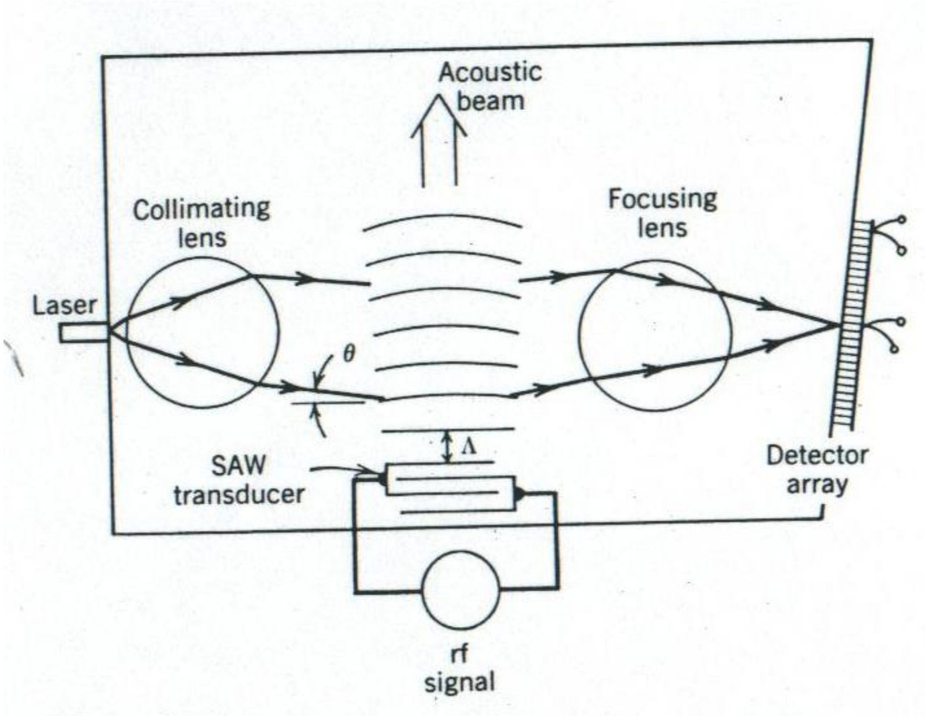
$$\delta y \approx \delta(x \tan\theta) \approx \delta(x\,\theta) \approx x\,\delta\theta = x\,\frac{\lambda}{W} \tag{14}$$

The angular spread (Δθ) due to the RF bandwidth results in a spatial separation, Δy (= x Δθ) of the frequency band at the photodetector array. Therefore the number of resolvable spots is the ratio of Δy to the minimum spot δy (note that x's are canceled out):

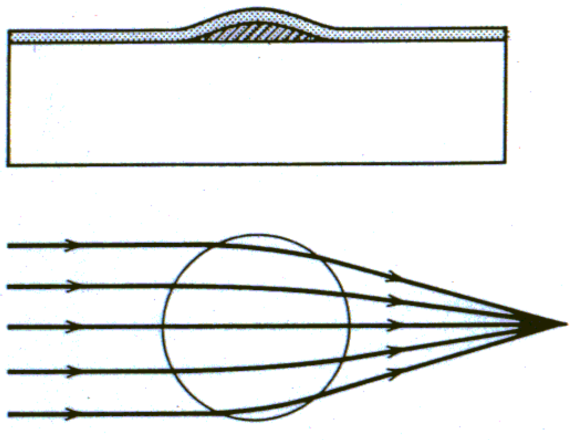
$$\#\text{ of resolvable spots} = \frac{\Delta y}{\delta y} = \frac{x\,\frac{\Delta f\,\lambda}{V_s}}{x\,\frac{\lambda}{W}} = \frac{\Delta f\,W}{V_s} = \Delta f\,\tau \tag{15}$$

An implementation of the guided-wave Bragg deflection for spectrum analysis is shown in the figure below. A semiconductor laser is butted up against one end of a dielectric waveguide and its radiated power is coupled into the film. The laser aperture is quite small so that the beam diverges rapidly. The beam is collimated using one of the three types of lenses as shown in the following figures. To understand how these lenses work we note that a lens has the property of introducing a phase shift for paraxial rays passing through it which decreases quadratically with distance away from the lens axis. For planar lens, this can be achieved as follows: in the Luneburg lens (a), the phase shift is controlled by placing a thicker layer of high-refractive-index materials near the center of the lens, the thickness decreasing quadratically toward the lens perimeter; in the geodesic lens (b), a spherical depression is introduced into the waveguide, rays propagating through the central lens regions must travel over a longer path than those near the periphery (the effective lens for all rays is the same); in the Fresnel lens (c) , use is made of the fact that it is the not the absolute phase shift through the lens that matters but only its value to within a multiple of 2π. Therefore, zones of phase shift alternating by π are produced by introducing a higher reflective-index film of appropriate spacing above the waveguide.

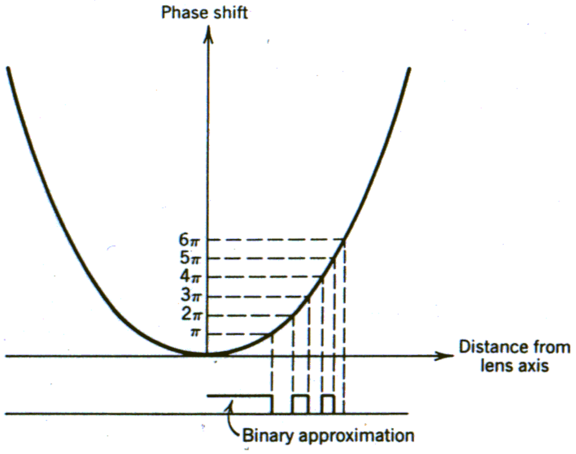
The collimated radiation from the output of the planar lens is next incident on the acoustic grating. A second lens focuses the deflected beam onto one of a number of detectors who can be butt coupled to the output waveguide edge.

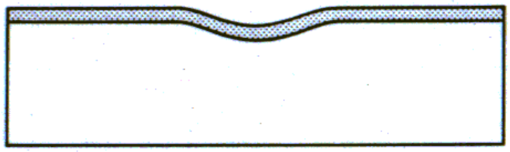


Acousto-Optical Spectrum Analyzer

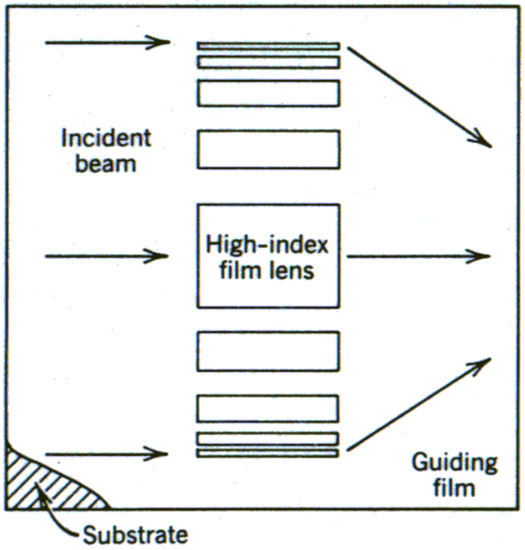


(a)


Phase shift
6π
5π
4π
3π
2π
π
Distance from lens axis
Binary approximation



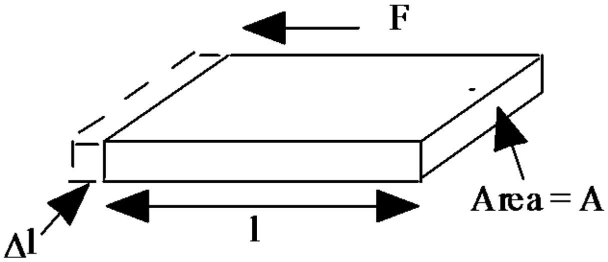
(b)


Incident beam
High-index film lens
Guiding film
Substrate

(c)

8.6 Photoelastic Effect

Distortion in a crystal is caused by the compressions and rarefractions of an acoustic wave that creates a perturbation in permittivity. Let's say a force F is applied to a bar with length l.



By Hooke's Law, the strain is proportional to the pressure applied at the end of the bar.

$$\frac{u}{l} = \frac{1}{c_o}\frac{F(u)}{A} = s \tag{16}$$

where u is the displacement, c_o is the elastic constant (for simplicity only, it should be a tensor). Let's consider the change in energy. The differential work dW in stretching the bar by du is,

$$dW = F(u)\,du = \frac{Ac_o}{l}\,u\,du \tag{17}$$

So the total work to stretch the bar from l to l + Δl is

$$W = \frac{Ac_o}{2l}\Delta l^2 = Al\,\frac{c_o}{2}\left(\frac{\Delta l}{l}\right)^2 = Al\,\frac{c_o}{2}\,s_o^{\,2} \tag{18}$$

and the energy stored per unit volume is

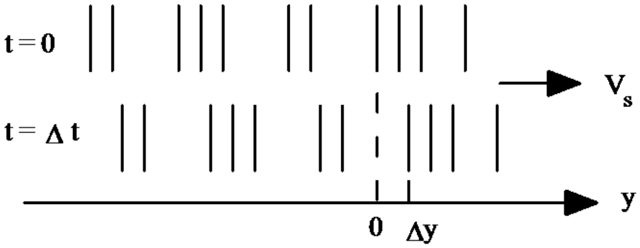
$$\frac{W}{Al} = \frac{c_o}{2}\,s_o^{\,2} \tag{19}$$

Suppose we launch an acoustic wave with a cross-sectional area A and traveling with an acoustic velocity V_s. The figure below shows a snapshot of the leading edge of the wave at times separated by Δt. During this period, all the energy stored in a volume V = A Δy is transferred to the right of the plane y= 0. Therefore the rate of transfer of acoustic energy to the right (acoustic power flow) is, using, Eq. 19,

$$\frac{\Delta W}{\Delta t} = P_{ac} = \frac{c_o}{2} s_o^2 \frac{(A\,\Delta y)}{\Delta t} \tag{20}$$

or

$$P_{ac} = \frac{c_o}{2} s_o^2 V_s A$$



Therefore the material strain is related to the acoustic power by,

$$s_o \propto \sqrt{\frac{P_{ac}}{A}} \tag{21}$$

If we further assume that the change in material optical permittivity is linearly proportional to the applied strain, we get,

$$\frac{\Delta\varepsilon}{\varepsilon_o} = \beta \sqrt{\frac{P_{ac}}{A}} \tag{22}$$

The constant β is dependent on the material's acoustic and so-called photoelastic properties. Photoelasticity is simply a measure of the change in optical permittivity of the material being strained. The value of β is also a function of the propagation directions of the optical signal and directions of the particle motion during acoustic disturbance.