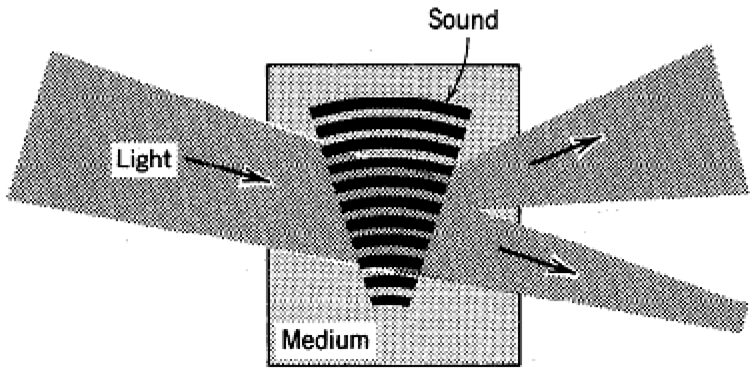
Acousto-Optic Modulation

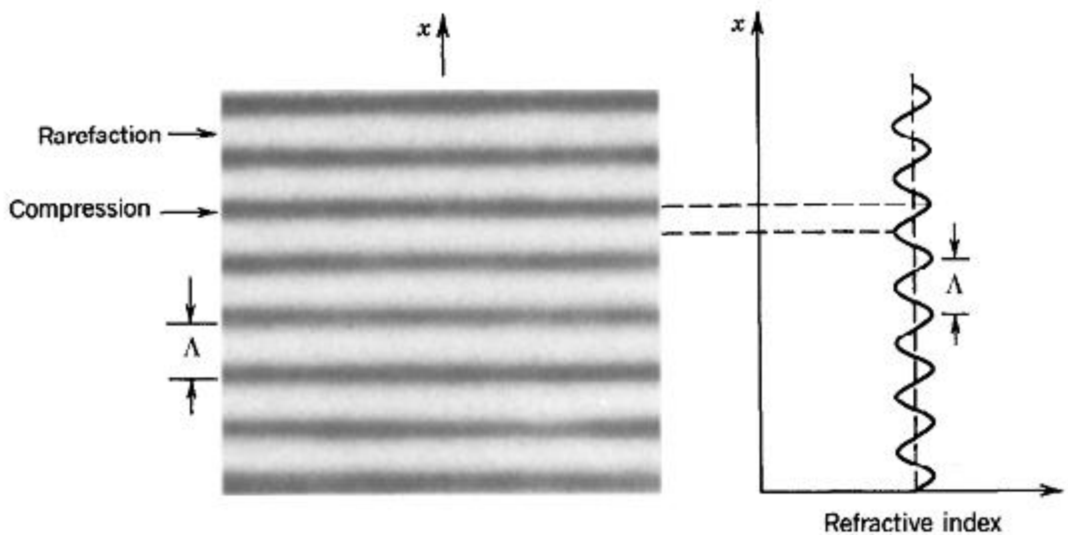
Reference: see B. E. A. Saleh's Fundamentals of Photonics

Acousto-optics is the phenomenon where light is diffracted by acoustic waves

Sir William Henry Bragg (**1862–1942**, left) and **Sir William Lawrence Bragg** (**1890–1971**, right), a father-and-son team, were awarded the Nobel Prize in 1915 for their studies of the diffraction of light from periodic structures, such as those created by sound.

The refractive index of an optical medium can be altered by the presence of sound.





Variation of the refractive index accompanying a harmonic sound wave.

A sound wave causes a sinusoidal perturbation in the density of the material, which shows up as a strain wave traveling at the velocity of sound, V_s, across the materials.

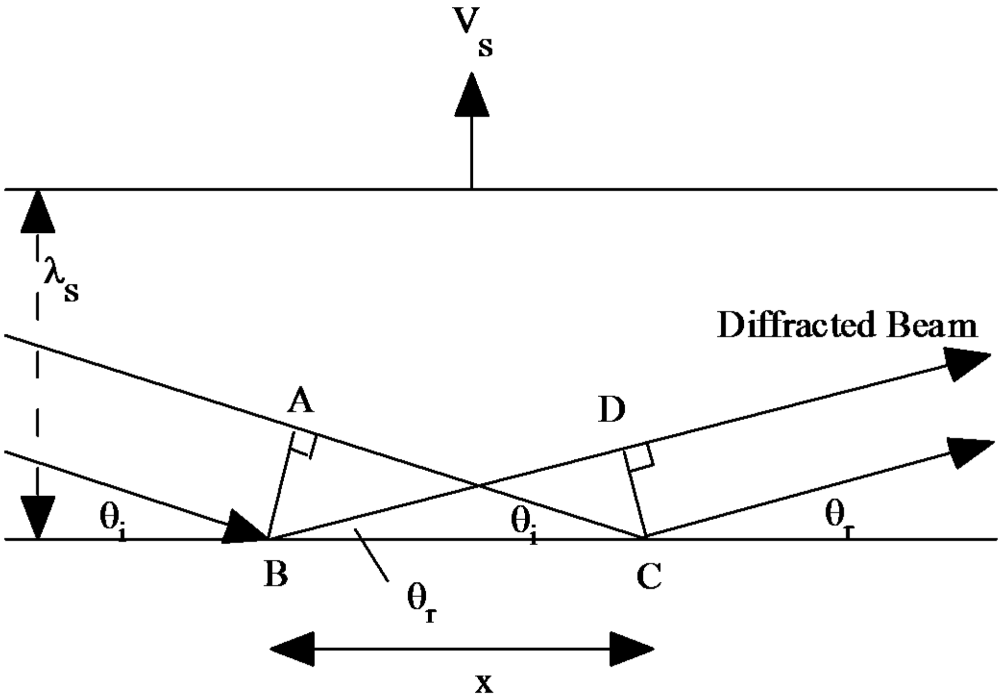
The change of density corresponds to a change of index, Δn, which is responsible for the modulation of a propagating electromagnetic wave.

$$\Delta n(r,t) = \Delta n \cos(\omega_s t - k_s z) \qquad (1)$$

Since the “waves” in an optical beam move very much faster than that of the sound wave, the index modulation $\Delta n(z,t)$ created by the sound wave appears stationary over many cycles of the optical beam.

So the modulation effect of the sound wave resembles that of a “Fix Grating” (which is actually moving).

We can characterize the sound waves as series of partially reflecting mirrors, separated by the sound wavelength λ_s, that are moving at the velocity V_s.
For the diagram below, if B, C represents two equal reflection points at the crystal, i.e. the index change is the same at both points:



For strong diffraction to occur in a given direction, all these points in the mirror contribute in phase to the diffraction along this direction.

Therefore, we need the path difference between two beams equals to an integral multiple of the wavelength in the medium, or since the path difference is AC-BD:

$$AC - BD = m\frac{\lambda}{n} \qquad (2)$$

where m is an integer, n is the medium index. Since:

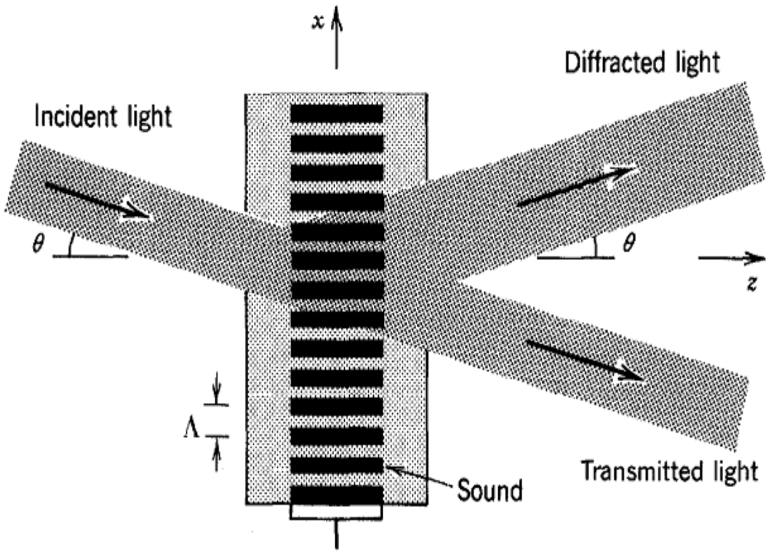
$$AC = x\cos\theta_i \;,\; BD = x\cos\theta_r \qquad (3)$$

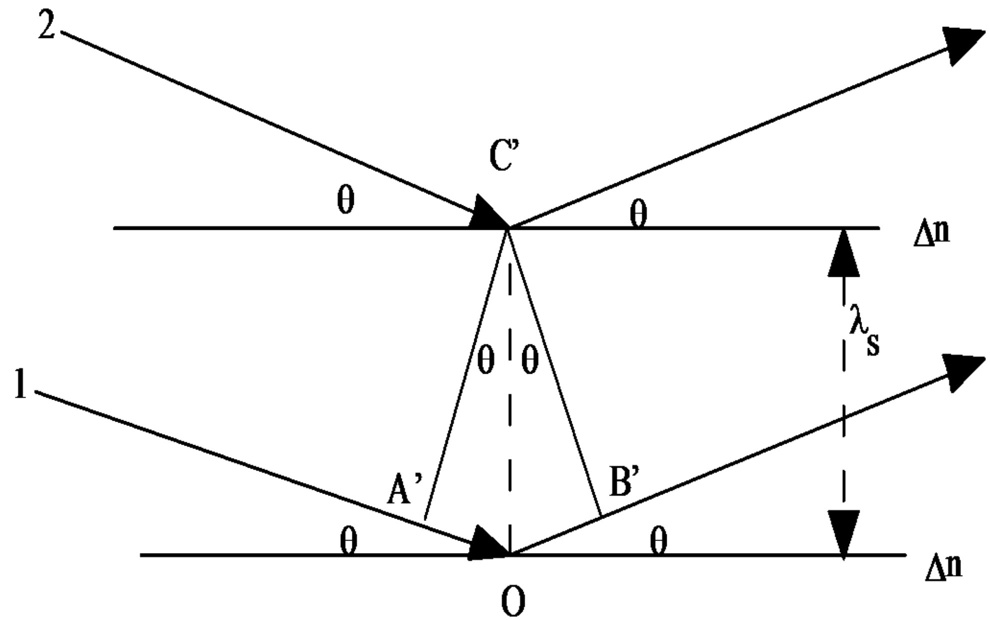
therefore:

$$x(\cos\theta_i - \cos\theta_r) = m\frac{\lambda}{n} \qquad (4)$$

Since x is arbitrary, the only way this works for all points in a given reflector is that we have the trivial case of m = 0, $\theta_i = \theta_r$ (this is the law of reflection).

We also require that the diffraction from any two acoustic phase fronts add up in phase along the direction of the reflected beam:





Note that the incident beam has a frequency ω, the diffracted beam has a frequency (ω + ω_s, where ω_s is the frequency of the moving sound wavefront at velocity V_s)

Here A'O + B'O is the additional distance traveled by beam 1. In order to obtain a constructive interference between the two beams:

$$A'O + B'O = m' \frac{\lambda}{n} \tag{5}$$

From the diagram we can see: $A'O = \lambda_s \sin\theta$

So Eq. 5 becomes:

$$2\lambda_s \sin\theta = m' \frac{\lambda}{n}$$

$$\sin\theta = \frac{\lambda}{2\Lambda}, \tag{6}$$

For the first order diffraction, m' =1, $\theta = \theta_1$, Eq. 6 is known as the Bragg Diffraction condition. (The second order and so on can occur for thin samples)

Example:

For sound wave with $V_s = 3 \times 10^5$ cm/s, a frequency (ν_s) of 500 MHz, and an optical wavelength of 0.5 μm (λ_{opt}/n).

This corresponds to a grating period (spacing) of $\lambda_s = V_s/\nu_s = 6$ μm.

We can use Eq. 6 to find that $\sin(\theta) = \lambda_{opt}/(2n\lambda_s) \sim 0.5/12$, so $\theta \sim 3.5^o$ is indeed very small.

Particle Picture of Bragg Diffraction of light

Photon, having the nature of wave, can be represented by a propagation vector k and so its phase at a given point x takes the exp i(ωt + k.x) dependence. In this representation, the incident photon has momentum $\underline{h}k_i$ (as a particle), and energy $\underline{h}\omega_i$; the diffracted photon has momentum $\underline{h}k_d$, and energy $\underline{h}\omega_d$. The sound 'wave' has momentum $\underline{h}k_s$, energy $\underline{h}\omega_s$. The interaction of the three particles involves an integration like this over the medium (question why integrate?):

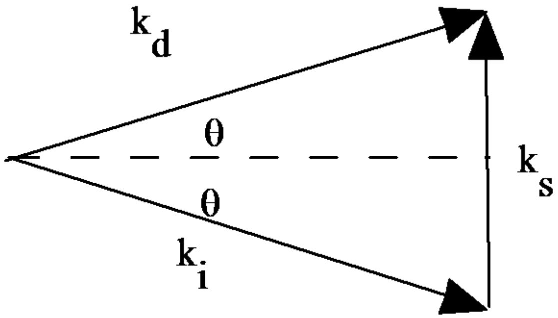
$$\int d^3r\, e^{-i(\omega_d t + k_d x)}\, \Delta n(z,t)\, e^{i(\omega_i t + k_i x)} \qquad (7)$$

To get nonzero integration we need to have a zero exponent:

$$k_d = k_i \pm k_s \qquad (8)$$

$$\omega_d = \omega_i \pm \omega_s \qquad (9)$$

Since ω_i and ω_d are much larger than ω_s, therefore $\omega_i \sim \omega_d$ in value and the magnitude of k_i and k_d are nearly the same (but k_s is not small).



This gives $k_s = 2\, k \sin\theta$ which is identical to Eq. 6 with m' =1. For the long interaction length case, i.e., the medium is large in extent, only the m=1 solution survives.

Coupled Wave (mode) analysis of A-O modulation

Recall the displacement vector D is:

$$D = \varepsilon_o E + P = \varepsilon E \qquad (10)$$

where P is the polarization of the medium. For simplicity, we assume the average index of the medium is close to that of vacuum.

$$\begin{aligned} P = (\varepsilon - \varepsilon_o) E = \varepsilon_o \left(\frac{\varepsilon}{\varepsilon_o} - 1\right) E &= \varepsilon_o (n^2 - 1) E \\ &= \varepsilon_o (n+1)\, \Delta n\, E \approx \varepsilon_o (2n)\, \Delta n\, E \\ &\approx 2\sqrt{\varepsilon \varepsilon_o}\, \Delta n(r,t)\, E(r,t) \end{aligned} \qquad (11)$$

where E is the sum of the two electric fields, E_i and E_d, and $\Delta n(r,t)$ is as before:

$$\Delta n(r,t) = \Delta n \cos(\omega_s t - k_s z) \qquad (1)$$

P can be regarded as the agent that facilitates the power exchange between E_i and E_d each of which has to satisfy the wave equation separately.

$$\nabla^2 E_{i\,or\,d}(r,t) = \mu\, \varepsilon \frac{\partial^2 E_{i\,or\,d}}{\partial t^2} + \mu \frac{\partial^2 P_{i\,or\,d}}{\partial t^2} \qquad (12)$$

From the wave equation:

$$\nabla^2 E_i(\mathrm{r,t}) = -\frac{1}{2}\left(\mathrm{k_i}^2\mathrm{E_i} + 2\mathrm{i\,k_i}\frac{\partial \mathrm{E_i}}{\partial \mathrm{r_i}} + \nabla^2\mathrm{E_i(r)}\right)e^{i(\omega_i t - k_i . r_i)} + \mathrm{c.c.} \quad (14)$$

The third term inside the bracket is slowly varying and can be ignored, and in the absence of the sound wave perturbation:

$$k_i^{\ 2} = \omega_i^{\ 2}\mu\varepsilon_o \qquad (15)$$

Using this results, we got:

$$\mathrm{k_i}\frac{\partial E_\mathrm{i}}{\partial r_\mathrm{i}} = \mathrm{i}\mu\frac{\partial^2 P_i}{\partial t^2}e^{-i(\omega_i t - k_i . r_i)} \qquad (16)$$

However, From Eqs. 11, 1, and 13, we have:

$$\mathrm{P_i} = \frac{1}{2}\sqrt{\varepsilon\varepsilon_o}\,\Delta n\ \mathrm{E_d}\left(e^{i(\omega_s + \omega_d)t - i(k_s + k_d).r}\right) + \mathrm{c.c.} \qquad (18)$$

which leads to:

$$\frac{dE_i}{dr_i} = -i\,\eta_i\,E_d\,e^{\,i(k_i - k_s - k_d).r} \qquad (19)$$

where $\eta_i = \frac{1}{2}\omega_i\sqrt{\mu\varepsilon_o}\Delta n$

Similarly:

$$\frac{dE_d}{dr_d} = -i\,\eta_d\,E_i\,e^{\,-i(k_i - k_s - k_d).r} \qquad (20)$$

where $\eta_d = \frac{1}{2}\,\omega_d\sqrt{\mu\varepsilon_o}\Delta n$

Eq. 19 and Eq. 20 are the coupled mode equations for this acousto-optic system