Lab 3 - Digital Baseband Data Link (two weeks)

February 4, 2007

Goal

Characterizing an antipodal baseband digital data link.

Pre- lab

Reading for week 1

1. ZT 7.1, 7.2.
2. Download your own personal copy of the following paper from IEEE Explore. (We cannot post this paper on the Web site because of copyright issues. Also note that you must be in the UCSD domain for the link to work.) Read Sections 1-2.
3. ZT 7.5 pages 365-366 on the raised cosine function that is used for spectral shaping. Additional reading on pulse shaping is provided in this tutorial.

Reading for week 2

1. Download and read the following paper on simulating communication links using Matlab.
2. Download and read the article on timing recovery. This article covers the combination of spectral shaping, matching filtering and timing recovery.

   An IEEE reference to the Gardner algorithm discussed in the paper is here. Details about interpolation for timing recovery are in a set of papers Part 1 and Part 2. (Note: must be in UCSD domain for link to these references to work.)

Problems and Simulations -Week 1

1. *PRBS*

   a) Determine the Fourier series, the autocorrelation function (Eq. 1 in the first IEEE paper), and the power spectrum for the 15 bit PRBS sequence shown in Figure 1 of the paper on PRBS.

b) Show that the envelope of the coefficients for the power spectrum is given by

$$P(f) \propto [\text{sinc}(\pi f T)]^2$$

where T is the period of the symbol.

c) Does the power spectrum depend on the order of the "blocks" of marks in the PRBS pattern? In other words, the pattern in Figure 1 in the paper has a block of four marks followed by a block of one mark, a block of two and another block of one. If the same blocks of marks were put in a different order (with spaces between the blocks) within the 15 bits, would the power spectrum change? Why?

2. *Spectral Shaped Waveforms*

The general raised cosine waveform may be written as

$$f(t) = \sum_{n=1}^{L} a_n p_{RC}(t - nT)$$

where a_n is the PRBS sequence and $p_{RC}(t)$ is given by Eq. (7.123) in ZT where T is the *symbol* period. Note that this is a *periodic* waveform with period LT where $L = 2^n - 1$ is the length of the PRBS.

a) Plot the waveform over one period of the PRBS for $\beta = 0$, 0.5, and 1. (The plots should be the same as in lecture.)

b) Determine the DC power in the waveform in dBm.

c) Using the FFT code from Computer Example 3.1, plot the power spectrum of the signal for each value of β. Be sure to include the 50 Ω resistor as that is the power that *Cal_SA* measures.

d) Determine the -3 dB bandwidth of the waveforms for $\beta = 0$ and $\beta = 1$.

Problems and Simulations -Week 2

1. *Antipodal baseband systems*

a) What is the correlation coefficient ρ for antipodal signaling? (See ZT)

b) What is the expression for P_e in terms of $z = E_b/N_0$ assuming a matched filter?

c) Let the received pulse be rectangular with amplitude A and period T and let the receiver filter be triangular with an impulse response $h(t) = 1 - 2|t|/T$. Determine an expression for P_e. (Note that because the filter is not matched, you must use the general expression listed in Eq. 7.31 in ZT.)

2. *Threshold error*

a) Using the expression for the threshold error given in Eq.(**??**), derive a modified expression for the probability of error. (Note the two contributions $P[E|s_1(t)]$ and $P[E|s_2(t)]$ are no longer equal.)

b) For low error rate systems, only one term in the result from part (a) is significant. For this case show that

$$P_e = \tfrac{1}{2}\, Q\left(x_0(1 - |\delta_z|)\right)$$

where $x_0 = \zeta/2$ is the argument to the Q function (See Eq. 7.53) in the absence of a threshold error, and δ_z is defined in Eq.(**??**).

3. *Clock offset error*

a) Accounting for the fact that there are four possible transitions for each data symbol ($0 \to 0$, $0 \to 1$, $1 \to 0$, $1 \to 1$) and that only two of them produce a clock offset error (see the background material), derive an expression for P_e in the presence of a clock offset error $\delta_\tau = \tau/T$ where τ is the absolute offset error, and T is the symbol time.

b) If $x_0 = \zeta/2 = 3$ is the argument to the Q function which corresponds to $P_e \approx 1.3 \times 10^{-3}$ and using the results of Problem 2, and the Problem 1(c) of from last week's Prelab, what is the relative change in the error rate for the maximum offset error if 8 samples per symbol is used? (Note that this error is typically corrected using interpolation techniques.)

4. *Simulation of the BER of a baseband data link*
We now follow the procedure in the paper "Bit-Error-Rate Simulation Using Matlab", to simulate a baseband digital link that uses antipodal keying.

a) If we wish to simulate a link to a specified BER, how many errors must we detect so that variance in the sample mean, which is our estimate of the BER, is less than 10% of the underlying randomness σ_X of the "true" BER distribution. (See Lab 2 Prelab.) Is this consistent with the number of samples chosen (without justification) in the paper?

b) If we wish to simulate to an error rate of $P_e = 10^{-4}$, approximately how many symbols must generated to produce the number of errors required by part (a)?

c) At a data rate of 1 Mb/s, how long will it take to generate the number of errors determined in part b)?

d) Simulate the link using only a single sample for the signal and noise per symbol. For the data, you will need a method to generate a random number that is ± 1 with equal probability. To do this, we will use the following Matlab code

```
data = sign(rand - 0.5);
```

The function `rand` generates a uniform random variable in the range $[0, 1]$. Subtracting 0.5 produces a random variable that is equal probable positive or negative. The `sign` function maps negative numbers to -1 and positive numbers to 1 producing an equal probable antipodal data stream.

For the noise replace the function **randn** with **normrnd(0, sigma,[1 N])** used in Lab 2. Use the same values of E_b/N_0 as in the paper. For the simulation of the

e) Using a `while` loop structure with the test being if you have collected enough errors based on the results of part (a), vary σ and thus E_b/N_0. simulated results along with the theoretical curve from Question 1(b) of this week's questions.

f) Simulate the link using 8 samples per symbol with the receiver integrating the samples over a symbol period. Compare with part (c) and comment. (Note that because you are integrating over the symbol interval, you will have to adjust the value of **sigma** in the noise routine to keep the same effective value of E_b/N_0 .

g) Offset the ideal sampling time by two samples and plot out E_b/N_0. Compare with the results of Problem 3(a).

Lab - Week 1

1 Pseudo random binary sequences (PRBS)

1.1 Up sampled raised cosine waveforms

In this section, we use a simulation in LabView to examine the temporal and spectral waveforms of up-sampled waveforms. In the next section, we create the VI to do this.

1. Open the *RC_sim* VI. This VI simulates up sampled un-filtered and raised cosine filtered waveforms.

2. Set the system filter to "none" don't use zeros and upsample by 16. This will produce a ± 1 bit pattern. Measure the power near DC and at the peak of the next lobe using a set of "free" cursors on a dB scale. Force the vertical range to be -60 to 0 dB and the horizontal to be 1-2 M. Save the spectrum.

3. Repeat using zeros. The waveform should be triangular with the ratio of the "DC" value of the envelope to the next peak should be different than in Step 2. Save the spectrum.

4. Now turn on raised cosine with $\beta = 0$ which corresponds to a sinc pulse. Save the spectrum. Notice that at each sampling point the value of the waveform is ± 1 because the pulses do not interfere. Change β and confirm this is true for any value of β.

5. Set $\beta = 1$ and change the up sampling factor. Note that while the number of samples of time waveform is increased, the spectrum does not change. Save the spectrum.

1.2 Generation of PRBS

In the first week of Lab, we will create a VI that generates an antipodal PRBS sequence with N samples per symbol. This VI will be used to characterize the baseband link in the second week of lab.

1. Create a new VI called PRBS and load the MT Generate Bits VI from the <addons> palette >> then <modulation> then >> <digital> in *LabView.*

2. Create front panel controls for PRBS sequence order, and total bits. The total bits should be ($2^L - 1$ where L is the pn sequence order. You can use the numeric palette to create this value.

3. Using math functions, subtract off 0.5 to produce an antipodal waveform that varies from ± 0.5V.

4. Up sample the PRBS sequence by a factor of 8 so that every symbol has 8 samples. The TA will provide the *UpSample* VI to do this.

5. Filter the upsampled sequence using the *RaisedCosine* VI. The output of the filtered, upsampled sequence will be a superposition of raised cosine pulses each with 8 samples per symbol.

6. Copy the code developed for noise generation from Lab 2 to create a waveform that is a PRBS waveform with additive noise. Use a value of $\sigma = 0.1$.

7. Connect the output waveform to both a time and spectral waveform display using the *FFT Power Spectrum*. Note that this VI does not calculate the power by dividing by 50 Ω nor does it display the power in dBm.

8. Set the PRBS order equal to 5 (31 bit sequence), the seed value = -1. The spectral waveform should similar to the one determined in Prelab which has a $\text{sinc}^2(x)$ envelope. Measure the power in the first harmonic using a set of cursors.

9. Determine the frequency spacing between the spectral lines as well as the spacing between the nulls.

10. Connect the output waveform to the sub-VI *Arb_INIT*. This VI will produce the analog waveform at the output of the AWG.

11. View the output waveform on the scope. Adjust the time base until you have a "stationary" waveform. Are you seeing the complete sequence?

12. Now connect the output of the AWG to the digitizer. Set the filter to "none" and measure and dump the generated spectrum using *Cal_SA*. Does it agree with the simulation for the un-filtered PRBS? Is the power measured with *Cal_SA* consistent with the un-calibrated power measured using the *FFT Power Spectrum* VI?

13. Now change the filter to a raised-cosine and view the power spectrum. Dump the spectrum from *Cal_SA* for $\beta = 0$, 0.5 and 1.

1.3 Eye Patterns

An eye-pattern is a visual representation of the data transitions. In this section, we generate an eye-pattern for the raised cosine waveform and examine the effect of β on the shape of the eye and the optimal sampling time.

1. Set the order of the PRBS to 5 and β=1.

2. Open *EyeDiagram.vi* and fill in the following inputs that should match the generation VI: samples per symbol, symbol rate.

3. Select the eye diagram tab and make sure that the whole eye is in the graph by selecting autoscale.

4. Estimate the minimum and maximum voltage values and use these values into the input for the histogram VI for the "max" and "min" values. Select a histogram size at 1000.

5. We will now "scan" the eye. To do this, use the "nth sample (0 - samples per symbol)" control and step through the values from zero to the value you specified under "samples per symbol". Display the histogram for each value. You should see a gradual change in the histograms as you go through the range of values, seeing either two, three, or four distributions on your histogram (and value of β).

6. Record $\zeta = (s_1(t) - s_0(t))/\sigma$ (ZT Eq. 7.32) where $s_1(t)$ is the mean value of the mark distribution at time t, $s_0(t)$ is the mean value of the space distribution at t and σ is the rms value of the noise for each of the sampling times. These *measured* values are given in boxes on the front panel of the VI. Find the sampling time that produces the largest value of ζ. Save the eye trace and histograms at this optimal time.

7. Repeat for $\beta = 0.5$ and $\beta = 0$(sinc pulses)

   a) How sensitive is ζ to the sampling time for each value of β?

   b) Which value of β causes ζ to be most sensitive to the sampling time.

   c) Which is the least sensitive? Why?

8. Set the transmit filter to "none" and examine the eye.

   a) How sensitive is ζ to the sampling time for the un-filtered case relative to the raised cosine pulses?

   b) Is the value of ζ at the optimal sampling time different for the unfiltered case relative to the filtered case?

1.4 Low-Pass Filtering

We will now filter the PRBS using a low-pass filter to compare the performance with a raised cosine filter and the un-filtered sequence.

1. Set the transmitter filter to "none" and insert a 2.5 MHz analog low-pass filter at the output of the AWG. Repeat steps 4-6 of Section 1.3 for the following data rate values: 1.25 Mb/s, 2.5 Mb/s, 5 Mb/s. Note that these correspond to values of 0.5, 1 and 2 for the simulations of the low-pass filtered signal shown in lecture.

   a) How does the value of ζ change as the data rate changes? Why?

   b) How sensitive is the peak value of ζ for the low-pass filter with respect to the raised cosine filter or no filter?

Lab - Week 2

In this week, we compare the analytical results from 154b with those of the Matlab simulation and of the measured BER in lab.

2 Baseband data generation and analysis

Here we measure the error rate of a baseband antipodal link under a variety of conditions.

2.1 BER vs. $E_b/N_0$for an unfiltered data waveform

When a matched filter is not used, the error rate can no longer be expressed in terms of E_b/N_0. We examine this situation in this section

1. Open *Baseband.vi* which is the main analysis block for baseband antipodal keying. Use 1 Msamples/s and a 31 bit PRBS with 8 samples per symbol.

2. Connect the AWG output to the Digitizer input. Turn off all filters.

3. Adjust the value E_b/N_0 until the estimated BER is $> 10^{-2}$.

   Because we are not using a matched filter, the value of E_b/N_0 on the front panel will not produce the same error rate as using a matched filter. Determining the correction factor is asked in Post lab.

4. Collect 100 errors and record both the true error rate (errors/transmitted symbols) as well as the error rate predicted from the mean and variance of the measured histograms displayed on the VI.

5. Repeat for an additional 9 values of E_b/N_0 with the last (highest) value producing an error rate of $<10^{-4}$. (Note that the VI can process about 10k samples/s so to collect 100 error will require 1 Msymbols or about 100 seconds. If it is taking significantly longer than this, ask the TA for help.)

6. **Note: the results of this step depend on which digitizer is used**. **The data sets will be exchanged so you have a data set from each digitizer**. Using the same value for E_b/N_0 that produced an error rate of $> 10^{-2}$, repeat for a $2^{12} - 1$ sequence running at 100 kSamples/sec. Depending on which digitizer is used, the error rate wil be significantly different than with the $2^5 - 1$ sequence running at 1 Msamples per second even though E_b/N_0 is the same. The explanation of this result is required for the lab write-up. Be sure to get the data set from another group using a different digitizer (10 points of BER vs. E_b/N_0).

2.2 BER vs. E_b/N_0using a Matched Filter

Repeat the measurements of the last section using a matched filter. Again note that same value of E_b/N_0 will produce a lower error rate using a matched filter relative to not using one. You should have 10 data points in total for this part.

2.3 Threshold error

1. Using a matched filter, set E_b/N_0 so that $P_e \approx 10^{-4}$ with no threshold error.

2. Increase the threshold in fixed steps of 2% of the peak amplitude of the data waveform until the threshold error is 20% of the peak value. There should be 11 data points including the error rate without a threshold error. In all cases, be sure to collect at least 100 errors for the measurements to be statistically significant.

2.4 Clock offset error

1. Using a matched filter, set E_b/N_0 so that $P_e \approx 10^{-4}$ with no clock error.
2. Using the front panel display, record the error rate for offset errors in the range of 1-8 samples where the last value is an offset error of half a symbol interval. There should be nine data points

3 BER for a bandlimited system

Set the input and output pulse shape so that the waveform before sampling is a raised cosine with $\beta = 0.5$. Record BER vs. E_b/N_0 down to $P_e \approx 10^{-4}$. You should again collect 10 values each with at least 100 errors.

Note that the VI calculates $E_b$using the following algorithm

$$E_b = \sum_{i=1}^{N} r^2[i]/50\Delta t$$

where $r[i]$ is the waveform sample, $\Delta t = 1/f_s$, and N is the number of samples over a *symbol period* and not the complete waveform. This algorithm is accurate for the square-law pulse where all the energy resides within T but must be modified for raised cosine waveforms that have a significant fraction of their energy outside the bit period. For the waveform using $\beta = 0.5$, $p_{RC}(t)$, the fraction of the total energy *inside* a period T is given by

$$\frac{\text{energy within a time } T}{\text{total energy in waveform}} = \frac{\int_0^1 p_{RC}^2(t)dt}{\int_{-\infty}^{\infty} p_{RC}^2(t)dt} = 0.49$$

so that we must scale $E_b$as measured by the VI 1/0.49=2.03 to account for the fact that the VI only measures 49% of the total energy.

4 Intersymbol interference

When the waveform before sampling is not a raised cosine, then there is interference. The amount of additional power required to produce the same error rate in the presence of interference to the power required without ISI is called a power penalty. Here will will measure the power penalty assuming that the peak amplitude of low-pass filtered signal is the same as the raised cosine function accounting for filter loses.

1. Connect the analog low-pass filter to the AWG port.
2. Using the *Fgen* VI, generate a 100 kb/s square-wave pulse with a peak-to-peak amplitude of 1 V.
3. Measure the square-wave on the scope with and without the filter. Adjust the amplitude of the square-wave so that the peak amplitude after the filter is 1 V.
4. Using this scaling factor, generate a 31-bit PRBS at 5 Msymbol/s that has the same peak amplitude after the filter as the other signals.

5. Using the matched filter for the square-wave waveform, measure the error rate for 10 values of E_b/N_0 that produces a BER down to 10^{-3}. Note that for each measurement, you may need to adjust the sampling time to produce a minimum error. Also note that the value of E_b/N_0 is again only calculated over a time T and not over the complete waveform. The approximate scaling factor will be determined in Post lab.

6. Increase E_b/N_0 and determine the additional power required (called a power penalty) to produce the same error rate in the presence of ISI as the BER without ISI.

Post Lab

1. Using the saved plots from the simulation VI on raised cosine waveforms and up sampling, estimate the 3 dB bandwidth of the following signals:

   a) The unfiltered data waveform without upsampling
   b) The unfiltered data waveform with up sampling
   c) The raised cosine waveform with $\beta = 1$
   d) The raised cosine waveform with $\beta = 0$

   Compare the results with those of Prelab and comment the spectral efficiency of each waveform.

2. Explain the difference in the ratios of the DC power to the power in the next peak for the square-wave waveform using "1's" and the triangular waveform using "zeros".

3. Based off the values of ζ determined from the eye-diagram analysis, estimate the error rate for the following systems:

   a) Raised cosine. Also explain why the value of β does not matter.
   b) Low-pass with the data rate equal to the bandwidth of the filter.

   Which estimate is more accurate? Why?

4. Using the data from Week 2 Section 2.2, plot the measured BER vs. E_b/N_0 along with the theoretical curve and the simulation on the same graph for the square-wave waveform using a matched filter. Comment and discuss any differences between the three curves.

5. On the same curve, plot the BER vs. E_b/N_0 for the square-wave waveform with and without a matched filter. Quantitatively (i.e. provide a formula) that explains the shift of the curves with respect to one another.

6. On the same curve, plot the BER vs. E_b/N_0 for the square-wave waveform and the raised cosine waveform both using matched filters. Be sure to correct for the energy in the raised cosine waveform. Are they what you expected?

7. On the same curve, plot the BER vs. E_b/N_0 for the square-wave waveform using a matched filter and the low-pass filtered waveform using the same integrate and dump receiver. From

this curve determine the power penalty associated with ISI. Is it the same for all values of E_b/N_0? Why or why not?

8. Explain the results of Section 2.2 Part 6 recalling that two of the digitizers are AC coupled, with a low-frequency cut-off for the high-pass filter of approximately 7 kHz, while the third is DC coupled.

9. Plot the values recorded for P_e vs. the normalized threshold error δ_z in Section 2.3 . Compare with the theoretical results from Prelab.

10. Plot the values recorded for P_e vs. the normalized clock offset error δ_z in Section 2.3 . Compare with the theoretical results from Prelab.