ECE 161A PS#2. Solution

1. (a) Yes (b) No (c) Yes
(d) Yes (e) Yes. (f) no.

2. $\boxed{\text{For } y_1[n],}$
(a) Yes (b) no, es. $x_1[n]=u[n]$ and $x_2[n]=u[-n-1]$
will have the same output.
(c) Yes (d) Yes (e) Yes
(f) $y_1^*(n) = Y(-x[n]) = \begin{cases} -x[n]/12, & x>0 \\ 0, & \text{o.w.} \end{cases}$

$-y_1[n] = \begin{cases} -x(n)/12, & x<0 \\ 0, & \text{o.w.} \end{cases}$

$\Rightarrow y_1^*(n) \neq -y_1[n]$ ∴ non-linear.

$\boxed{\text{For } y_2[n]}$.
a) Yes (b) no (c) Yes (d) yes (e) yes (f) no.

3.

For $y_1[n]$ a)yes b)no c)yes d)yes e)no f)yes

For $y_2[n]$ a)yes b)no c)yes d)yes e)no f)no (why?)

4.

$$z(n) = x_1(n+5) * x_2[n-N]$$
$$= (x_1(n) * \delta(n+5)) * (x_2(n) * \delta(n-N))$$
$$= (x_1(n) * x_2(n)) * (\delta(n+5) * \delta(n-N))$$
$$= y(n) * (\delta(n+5-N)) = \underline{y(n-N+5)} \;//$$

another way:

let $z_1[n] = x_1(n+5)$, $z_2[n] = x_2[n-N]$

$$\underline{z[k]} = z_1[k] * z_2[k] = \sum_{n=-\infty}^{\infty} z_1[k-n]\, z_2[n] = \sum_{n=-\infty}^{\infty} x_1[k-n+5]\, x_2[\overset{u}{\widetilde{n-N}}]$$
$$= \sum_{u=-\infty}^{\infty} x_1[(k-N+5)-u]\, x_2[u] = \underline{y[k-N+5]} \;//$$

5.

(a) $y[n] = g_{ev}[n] \circledast h_{ev}[n] = \sum_{k=-\infty}^{\infty} h_{ev}[n-k] g_{ev}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{ev}[-n-k] g_{ev}[k]$. Replace k by $-k$. Then the summation on the left becomes $y[-n] = \sum_{k=-\infty}^{\infty} h_{ev}[-n+k] g_{ev}[-k] = \sum_{k=-\infty}^{\infty} h_{ev}[-(n-k)] g_{ev}[-k]$
$= y[n]$. Hence $g_{ev}[n] \circledast h_{ev}[n]$ is an even sequence.

(b) $y[n] = g_{ev}[n] \circledast h_{od}[n] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{ev}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{od}[-n-k] g_{ev}[k] = \sum_{k=-\infty}^{\infty} h_{od}[-n+k] g_{ev}[-k]$
$= \sum_{k=-\infty}^{\infty} h_{od}[-(n-k)] g_{ev}[-k] = -\sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{ev}[k] = -y[n]$.
Hence $g_{ev}[n] \circledast h_{od}[n]$ is an odd sequence.

(c) $y[n] = g_{od}[n] \circledast h_{od}[n] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{od}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{od}[-n-k] g_{od}[k] = \sum_{k=-\infty}^{\infty} h_{od}[-n+k] g_{od}[-k]$
$= \sum_{k=-\infty}^{\infty} h_{od}[-(n-k)] g_{od}[-k] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{od}[k] = y[n]$.
Hence $g_{od}[n] \circledast h_{od}[n]$ is an even sequence.

6. Refer to page 55 of text book