Brief Review: Sampling (Assumed from ECE101)

Sampling is used, for example, in A/D Conversion, although here we ignore the effects of *quantization*.

Discretize a continuous time signal for CDs, computers, etc.

Define the continuous time impulse train as:

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$x(t)$ is the continuous time signal we wish to sample

Let $y(t) = x_s(t) = x(t)p(t)$ be the sampled signal. Then,

$$x_s(t) = y(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t - kT) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t - kT)$$

$$X_s(\omega) = Y(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$

by the multiplication property of the continuous time Fourier Transform. $X(\omega)$ is the Fourier Transform of $x(t)$.

Now find the Fourier Transform of $p(t)$, the infinite impulse train:

$$P(\omega) = \mathcal{F}[\sum_{k=-\infty}^{\infty} \delta(t - kT)]$$

Use the Fourier Transform of periodic signals since the impulse train is a periodic signal (with $\omega_0 = \omega_s$)

$$\mathcal{F}[\sum_k a_k e^{jk\omega_s t}] = \sum_k 2\pi a_k \delta(\omega - k\omega_s)$$

Find a_k for the periodic impulse train:

$$\begin{aligned} a_k &= \frac{1}{T}\int_T x(t)e^{-jk\omega_s t}dt \\ &= \frac{1}{T}\int_T \delta(t)e^{-jk\omega_s t}dt = \frac{1}{T} \text{ for all } k \end{aligned}$$

Therefore

$$\begin{aligned} \mathcal{F}[\sum_{k=-\infty}^{\infty} \delta(t-kT)] &= \sum_k \frac{2\pi}{T}\delta(\omega - k\omega_s) \\ &= \sum_k \omega_s \delta(\omega - k\omega_s) \end{aligned}$$

Thus, an impulse train in time has a Fourier Transform that is an impulse train in frequency.

The spacing between pulses in time is T, and the spacing between pulses in frequency is $\frac{2\pi}{T}$.

So increasing the spacing in time decreases the spacing in frequency and vice versa. This is an important result!

Back to $X_s(\omega)$
Let $\omega_s =$ be the sampling frequency

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\begin{aligned}
X_s(\omega) &= \frac{1}{2\pi} X(\omega) * [\omega_s \sum_k \delta(\omega - k\omega_s)] \\
&= \frac{\omega_s}{2\pi} \sum_k X(\omega - k\omega_s) = \frac{1}{T} \sum_k X(\omega - k\omega_s)
\end{aligned}
$$

or we get replicated, scaled version of $X(\omega)$.

ω_b is for "bandwidth"

Now, what if $\omega_s - \omega_b < \omega_b$?

We would get overlap of the islands or "aliasing." Therefore, we need $\omega_s - \omega_b > \omega_b$ or $\omega_s > 2\omega_b$ to avoid aliasing.

Sampling Theorem says need $\omega_s > 2\omega_b$ to recover $x(t)$ from its samples–in other words, we need to sample at least twice the highest frequency to avoid aliasing.

We hear music up to $20kHz$ and CD sampling rate is $44.1kHz$.

A dog would need a higher quality CD since they hear higher frequencies.

You can recover $x(t)$ from $y(t)$ by using a low pass filter to recover the center island.