

Problem Set #4
Due: October 30, 2007

1. State the restriction on the variables a and b for the convolution sums to converge. If no restriction is needed, state no restriction.

- $a^n u[n] * b^n u[n + 6]$
- $a^n u[n] * b^n u[-n - 6]$

2. Show that for a causal system with impulse response $h[n]$, whose z-transform is $H(z)$ we have

$$h[0] = \lim_{z \rightarrow \infty} H(z).$$

3. A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z) = \frac{z^2 + z}{(z - 1/2)(z + 1/4)}.$$

- What is the ROC of $H(z)$?
- Is this system stable? Explain.
- Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\frac{1}{3}\left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3}2^n u[-n - 1].$$

- Draw the pole-zero plot for $X(z)$ and indicate its ROC.

4. Determine the inverse z-transform $x_1[n]$ of the following z-transform:

$$X_1(z) = \frac{1}{1 - z^{-7}} + \frac{1}{z}, \quad |z| > 1$$

HINT: use the "useful sum" formula and also results we discussed in page 22 of notes.

5. Given $x[n] = a^n u[n]$, find the Z-transform and ROC of $x[n/3]$ and $b^{2n} x[n]$.
6. Determine the difference equation that describes the inverse system to system H , where the transfer function H is given by

- $H(z) = \frac{z^3 - 1.8z^2 + 0.81z}{3(z-1)^2}$
- $H(z) = \frac{z^2 - 0.75z + 0.125}{2z^2 - 0.75z}$

7. Consider the Z-transform

$$H(z) = \frac{3z^3 + 49.2z^2 + 10.2z}{(z - 1)(z - 12)(z - .6)}$$

- Find all possible ROC for this function.
- Find the inverse transform for each region of convergence found above.
- You are told that $H(z)$ is the frequency response of a stable system; find its impulse response.