

Solution to problem 5 of HW3

5)

$$y[n] - 0.3y[n-1] - 0.04y[n-2] = x[n] + 2x[n-1]$$

for homogeneous solution let $y_h[n] = \lambda^n$. Thus the characteristic function is

$$\lambda^n - 0.3\lambda^{n-1} - 0.04\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.3\lambda - 0.04) = 0$$

$$\lambda^{n-2}(\lambda - 0.4)(\lambda + 0.1) = 0$$

$$\text{thus, } y_h[n] = c_1(0.4)^n + c_2(-0.1)^n$$

for impulse response, we let $x[n] = \delta[n]$ and $y_p[n] = k\delta[n]$ for $n > 0$

because for $n > 0$ $\delta[n] = 0$, thus our solution is only of the homogeneous one, i.e. $y[n] = h[n] = y_h[n]$.

$$\begin{aligned} n=0 : y[0] - 0.3y[-1] - 0.04y[-2] &= x[0] + 2x[-1] \\ y[0] + 0 \quad \quad \quad + 0 &= 1 + 0 \rightarrow y[0] = 1 \end{aligned}$$

$$\begin{aligned} n=1 : y[1] - 0.3y[0] - 0.04y[-1] &= x[1] + 2x[0] \\ y[1] - 0.3y[0] + 0 &= 0 + 2 \rightarrow y[1] = 2.3 \end{aligned}$$

solve for c_1 and c_2 by plugging $y[0]=1$ and $y[1]=2.3$ in $y[n] = c_1(0.4)^n + c_2(-0.1)^n$

we get $c_1 = 4.8$ and $c_2 = -3.8$