

UNIVERSITY OF CALIFORNIA, SAN DIEGO

Electrical & Computer Engineering Department

ECE 259BN - Winter Quarter 2008

Trellis-Coded Modulation

Problem Set #3 Due Thursday, February 14, 2008

Reading: Text (Schlegel and Perez), Chapters 6.1 - 6.3; 7.6 - 7.8; 10
Text (Biglieri), Chapter 2.7
Supplemental Text (McEliece), Chapters 9.1 - 9.3, Appendix D
Reserve papers: "Turbo Codes" papers 1 - 4

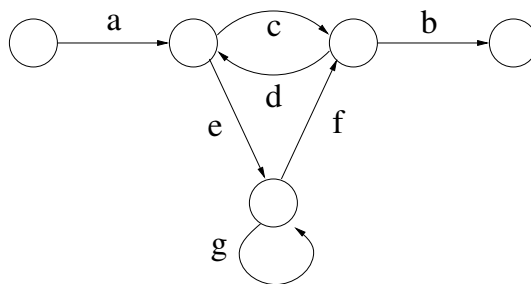
1. A calculus exercise for an appetizer: Using integration by parts, prove the following bound on the Q-function:

$$Q(x) \leq \frac{1}{x\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad x \geq 0.$$

Hint: Start by rewriting $Q(x)$ as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \left(\frac{1}{y}\right) ye^{-\frac{1}{2}y^2} dy.$$

2. Estimate as best you can the bit-error-rate performance as a function of signal-to-noise ratio for the "Planetary Standard" code, and compare your estimate to your simulation results (from P.S. #2, problem 4). Explain the method you used to get your answer.
3. (a) Using the graph reduction techniques discussed in class, determine the formula for the transfer function of the "split" 4-state graph shown below:



- (b) Use your answer to part (a) to determine the input-output weight enumerating function (IOWEF) for the rate 1/2, systematic encoder for our favorite degree 2, $d_{free} = 5$ code:

$$G(D) = \left[1, \frac{1 + D + D^2}{1 + D^2} \right].$$

- (c) For the encoder in part (b), find a minimum Hamming weight input that produces an output of minimum Hamming weight.

- (d) Now find the IOWEF corresponding to the following systematic encoder for a rate 1/3 code:

$$G(D) = [1, 1 + D^2, 1 + D + D^2].$$

- (e) Draw a trellis diagram for the rate 2/3 encoder with generator matrix:

$$G(D) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 + D^2 & 1 + D + D^2 \end{bmatrix}$$

Do you notice anything unusual about this trellis? Do you think this encoder generates a “good” rate 2/3 code?

4. An $(n, k) = (3, 2)$ convolutional code has systematic generator matrix

$$G(D) = \begin{bmatrix} 1 & 0 & 1 + D \\ 0 & 1 & D \end{bmatrix}$$

- (a) For this encoder, find a physical realization that requires the smallest number of delay elements. Show the corresponding shift-register circuit as well as the state diagram.
- (b) Compute the output-weight enumerator $T(D)$ and the complete path enumerator $T(D, L, I)$.
- (c) Determine the multiplicities of the 3 smallest codeword weights, and find the input words of smallest weight that can generate a codeword of each of these codeword weights.
- (d) Plot the estimates of the first-error probability versus E_b/N_o that you get by using the first 1, 2, 10, and 20 terms of the union bound we derived in class.
5. This problem will give you some insights into the effect of the interleaver (permuter) upon the distance spectrum of a turbo code. Consider a parallel concatenation of two rate 1/2, recursive, systematic convolutional encoders with feedback polynomial and feedforward polynomial given by $(3, 1)_{octal}$. The non-systematic portion of the constituent encoder is simply the rate-1 “accumulate” encoder $p(D)$ given by:

$$p(D) = \frac{1}{1 \oplus D}.$$

Assume the frame (interleaver) length is $N = 15$.

- (a) Compute the weight spectrum of the code generated by each of the following interleavers, and comment upon any differences you observe:
- (i) the “identity” interleaver;
- (ii) the hand-designed “Öberg” interleaver*:

$$\pi = [6, 2, 13, 4, 15, 3, 11, 12, 1, 14, 7, 10, 5, 9, 8];$$

*Mats Öberg was my second Ph.D. student. He designed the interleaver for this problem.

- (iii) another interleaver of your choice.

- (b) Consider the parallel concatenation of constituent codes from part (a) with a uniform interleaver. Compute the first 20 terms $a(w, z)$ of the average input-redundancy weight enumerating function (IRWEF),

$$A(W, Z) = \sum_{w=0}^N \sum_{z=0}^{2N} a(w, z) W^w Z^z,$$

for frame lengths $N = 10, 100,$ and 1000 . (The ordering on the coefficients is by increasing Hamming weight $d = w + z$, and by lexicographic order of indices among coefficients with the same Hamming weight d .)