UNIVERSITY OF CALIFORNIA, SAN DIEGO

Electrical & Computer Engineering Department

ECE 259BN - Winter Quarter 2008

Trellis-Coded Modulation

Problem Set #5 Due Tuesday, March 18, 2008

Reading: Text (Schlegel and Perez), Chapters 3.1 - 3.4
Text (Biglieri), Chapter 3
Reserves: Trellis-Coded Modulation papers 1,7,8 (Ungerboeck)
Wolf Notes, "Trellis-Coded Modulation"

1. In class, we derived the general *a posteriori probability (APP)* decoder for a **systematic** encoder. This problem asks you to modify that derivation to generate the general *APP* decoding algorithm for a **non-systematic** encoder.

2. We will compare the performance of coded QAM systems to that of uncoded QAM systems. To that end:

   (a) Derive the exact expression for the probability of symbol error for the standard 16-QAM constellation with minimum symbol distance d and with additive, complex-valued, zero-mean, Gaussian noise having independent real and imaginary parts, each with variance σ^2.

   (b) Derive the upper bound for the probability of symbol error for the square M-QAM constellation, $M = (2m)^2$, as a function of the average SNR per symbol $\gamma = E_s/2\sigma^2$, where E_s is the average symbol energy:

$$P(e) \leq 4Q\left(\left[\frac{3\gamma}{M-1}\right]^{1/2}\right).$$

3. In Ungerboeck's 1982 paper, the following statements are made:

   (a) For 8-PSK, $w(e_i) > \Delta_{q(e_i)}$ only for $e_i = 101$.

   (b) For 16-QASK, $w(e_i) > \Delta_{q(e_i)}$ only for $e_i = 1001, 1101, 1111$.

   Determine if these statements are correct. If not, give a counterexample. Justify your answers.

4. (a) Assuming that the average symbol energy is normalized to unity, $E\{|a_n|^2\} = 1$, derive the minimum symbol distance for the following constellations:

   (i) 8AMPM

   (ii) 16QAM

   (iii) 32AMPM

(iv) 64QAM

(b) Using the results of part (a), explain how the coding gains in Table III of Ungerboeck's paper are obtained from the normalized form of the free squared-Euclidean distance $d^2_{free}/\Delta_1^2(coded)$.

5. Rotational invariance – invariance of the received symbol sequence to phase shifts – is a very important property for practical TCM schemes to have. This problem explores some issues related to rotational invariance.

   (a) Prove that the encoder for the 4-state, 8-PSK Ungerboeck code (depicted on page 8 from G. Ungerboeck, "Trellis-Coded Modulation with Redundant Signal Sets, Part I: Introduction," *IEEE Communications Magazine,* vol. 25, no. 2, pp. 5–11, February 1987) can be incorporated into a TCM system that is invariant to 180° phase shift. In other words, devise a system such that if the coded sequence were rotated by 180°, the decoder would still produce the correct decoded data.

   (b) Recall the discrete-time model of the dicode recording channel introduced in Problem 3 of Problem Set #2. (Recall that the input-output relationship is: $y_n = x_n - x_{n-1}$.) Let the channel input sequence $\mathbf{x}$ produce channel output sequence $\mathbf{y}$.

      (i) What output does the detector/decoder produce if the channel output suffers a 180° phase shift?

      (i) Design a "precoder" – a shift-register circuit, possibly with feedback – and a "post-coder" that, together with the original system, provide invariance to 180° phase shift. Can you combine the decoder and "post-coder" into a more simply described operation?