University of California, San Diego

Electrical & Computer Engineering Department
ECE 259BN - Winter Quarter 2008
Trellis-Coded Modulation

Solutions to Problem Set 4

Please send comments to psiegel@ucsd.edu.

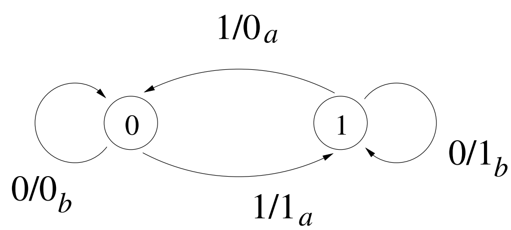
Problem 1

a)

Output	*Input Weight*				
Weight	0	1	2	3	4
0	1	0	0	0	0
1	0	1	3	0	0
2	0	1	2	2	1
3	0	1	1	2	0
4	0	1	0	0	0

b)

To help count all length-n paths of input weight w and output weight h, we consider an equivalent state graph in which the output symbols on zero-input loops are labeled with b.



Let 0_b^l denote a sequence of 0_b's of length l (possibly zero), and similarly for 1_b^l. From this state diagram, we see that all output sequences must have the form,

$$0_b^{r_1} 1_a 1_b^{s_1} 0_a 0_b^{r_2} 1_a 1_b^{s_2} 0_a \cdots 0_b^{r_m} 1_a 1_b^{s_m} 0_a 0_b^{r_{m+1}} ,$$

if w is even (where $m = \lfloor \frac{w}{2} \rfloor$), and

$$0_b^{r_1} 1_a 1_b^{s_1} 0_a 0_b^{r_2} 1_a 1_b^{s_2} 0_a \cdots 0_b^{r_m} 1_a 1_b^{s_m} 0_a 0_b^{r_{m+1}} 1_a 1_b^{s_{m+1}} ,$$

if w is odd. Conversely, any sequence of this form has a valid path through the state-diagram (from state 0). Notice the reason for two cases: when w is even the final state must be 0; when w is odd, it must be 1.

For any w, the zero sub-sequence of length $(n-h)$ has the form

$$0_b^{r_1} 0_a 0_b^{r_2} 0_a \cdots 0_b^{r_m} 0_a 0_b^{r_{m+1}} \,,$$

and we observe that the m 0_a's can occur anywhere among these $(n-h)$ positions. Equivalently, there are $\begin{pmatrix} n-h \\ m \end{pmatrix}$ possible sub-sequences of this form.

The case for a length-h sub-sequence of ones is slightly different. When w is even, we have

$$1_a 1_b^{s_1} 1_a 1_b^{s_2} \cdots 1_a 1_b^{s_m}.$$

The first one must always be 1_a. The other $(m-1)$ 1_a's can then be placed in any of the remaining $(h-1)$ positions, so there are $\begin{pmatrix} h-1 \\ m-1 \end{pmatrix}$ possible sub-sequences that can arise. When w is odd, we have an extra 1_a, and correspondingly $\begin{pmatrix} h-1 \\ m \end{pmatrix}$ sub-sequences. Note that these formulas can be incorporated into

$$\begin{pmatrix} h-1 \\ \lceil \frac{w}{2} \rceil - 1 \end{pmatrix},$$

which holds for both cases of w.

Finally, any combination of a valid zero sub-sequence and a valid one sub-sequence is a possible output sequence. Hence, the total number of output sequences is

$$\begin{pmatrix} n-h \\ \lfloor \frac{w}{2} \rfloor \end{pmatrix} \begin{pmatrix} h-1 \\ \lceil \frac{w}{2} \rceil - 1 \end{pmatrix}.$$

Problem 2

a)

For the parallel concatenation of p codes,

$$A_w^C(Z) = \begin{pmatrix} k \\ w \end{pmatrix}^{-p+1} \cdot \prod_{i=1}^{p} A_w^{(i)}(Z)\,,$$

which follows simply from results given in class. The IRWEF from class is equivalent to the IOWEF in this case, since no extra systematic bits are transmitted. Deriving this in terms of the coefficients, we find

$$A_{w,h}^C = \frac{\displaystyle\sum_{h_{p-1}=0}^{h} \sum_{h_{p-2}=0}^{h_{p-1}} \cdots \sum_{h_1=0}^{h_2} A_{w,h_1}^{(1)} A_{w,h_2-h_1}^{(2)} A_{w,h_3-h_2}^{(3)} \cdots A_{w,h-h_{p-1}}^{(p)}}{\begin{pmatrix} k \\ w \end{pmatrix}^{p-1}}$$

b)

For the serial concatenations, we repeatedly apply the formula given in class for a serial concatenation of two codes.

$$A^C_{w,h} = \sum_{h_{p-1}=0}^{n_{p-1}} \sum_{h_{p-2}=0}^{n_{p-2}} \cdots \sum_{h_1=0}^{n_1} \frac{A^{(1)}_{w,h_1} A^{(2)}_{h_1,h_2} A^{(3)}_{h_2,h_3} \cdots A^{(p)}_{h_{p-1},h}}{\binom{n_1}{h_1}\binom{n_2}{h_2}\cdots\binom{n_{p-1}}{h_{p-1}}}$$

c)

Step 1. Define C_5 as the parallel concatenation of C_3 and C_4.

$$A^{(5)}_{w,h} = \frac{\sum_{h_3=0}^{h} A^{(3)}_{w,h_3} A^{(4)}_{w,h-h_3}}{\binom{k_3}{w}}$$

Step 2. Define C_6 as the serial concatenation of C_1 and C_5.

$$\begin{aligned} A^{(6)}_{w,h} &= \sum_{h_1=0}^{n_1} \frac{A^{(1)}_{w,h_1} A^{(5)}_{h_1,h}}{\binom{n_1}{h_1}} \\ &= \sum_{h_1=0}^{n_1} \sum_{h_3=0}^{h} \frac{A^{(1)}_{w,h_1} A^{(3)}_{h_1,h_3} A^{(4)}_{h_1,h-h_3}}{\binom{n_1}{h_1}\binom{k_3}{w}} \end{aligned}$$

Step 3. C is the parallel concatenation of C_2 and C_6.

$$\begin{aligned} A^C_{w,h} &= \frac{\sum_{h_2=0}^{h} A^{(2)}_{w,h_2} A^{(6)}_{w,h-h_2}}{\binom{k_2}{w}} \\ &= \sum_{h_1=0}^{n_1} \sum_{h_2=0}^{h} \sum_{h_3=0}^{h-h_2} \frac{A^{(1)}_{w,h_1} A^{(2)}_{w,h_2} A^{(3)}_{h_1,h_3} A^{(4)}_{h_1,h-h_2-h_3}}{\binom{n_1}{h_1}\binom{k_2}{w}\binom{k_3}{w}} \end{aligned}$$